Beyond the Standard Model

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Abstract
Despite the success of the standard model in describing a wide range of data, there are reasons to believe that additional phenomena exist, which would point to new theoretical structures. Some of these phenomena may be discovered in particle physics experiments in the near future. These lectures overview hypothetical particles, solutions to the hierarchy problem, theories of dark matter, and new strong interactions.

1   What is the standard model?

The Standard Model of particle physics is an $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge theory with quarks transforming in the $q_L^i \sim (3, 2, 1/6)$, $u_R^i \sim (3, 1, 2/3)$ and $d_R^i \sim (3, 1, -1/3)$ representations of the gauge group, and leptons transforming as $L_L^i \sim (1, 2, -1/2)$ and $e_R^i \sim (1, 1, -1)$. The index $i = 1, 2, 3$ labels the generations of fermions. The Standard Model also includes a single Higgs doublet [1] transforming as $H \sim (1, 2, 1/2)$. The vacuum expectation value (VEV) of the Higgs doublet, $\langle H \rangle = (v_H, 0)$, breaks the electroweak $SU(2)_W \times U(1)_Y$ symmetry down to the gauge symmetry of electromagnetism, $U(1)_{\rm em}$. With these fields of spin 1 (gauge bosons), 1/2 (quarks and leptons) and 0 (Higgs doublet), the Standard Model is remarkably successful at describing a tremendous amount of data [2] in terms of a single mass parameter (the electroweak scale $v_H \approx 174$ GeV) and 18 dimensionless parameters [3].

If one requires that the Lagrangian contains only renormalizable interactions, then the Standard Model cannot accommodate gravity or neutrino masses. However, the requirement of renormalizability goes beyond the sound comparison of theory and experiment, by imposing theoretical constraints at energy scales well above those accessible in current experiments.

Gravity is nicely imbedded in an extension of the Standard Model through the inclusion of a graviton (massless spin-2 particle) with dimension-5 couplings to the stress-energy tensor [4] suppressed by the Planck scale $M_P = 2 \times 10^{19}$ GeV. Graviton exchange reproduces (up to order $1/M_P$ effects) general relativity, so this theory describes gravitational interactions with sufficient accuracy for practical purposes (issues related to quantum gravity are not experimentally accessible in the foreseeable future).[1]

Neutrino masses can be included in a couple of ways which may be differentiated in principle through future experiments [5]. An important dichotomy is whether the neutrino masses are of Majorana or Dirac type. Majorana masses may be obtained from dimension-5 operators of the type

$$\frac{c_{ij}}{M_N} H H \left(\overline{L^c}_L^i L_L^j \right) . \tag{1}$$

The dimensionless coefficients c_{ij} then determine the elements of the neutrino mass matrix through $m_\nu^{ij} = c_{ij} v^2 / M_N$. Imposing $c_{ij} < O(1)$, the measured atmospheric neutrino mass-squared difference $|\Delta m_{\rm atm}^2| \approx (0.05\,{\rm eV})^2$ requires the mass scale where the description in terms of dimension-5 operators breaks down to satisfy $M_N \lesssim 10^{14}$ GeV. The nonrenormalizable operators (1) may be generated by the tree-level exchange of gauge-singlet fermions (commonly called "right-handed neutrinos"[2] and labelled by ν_R) or $SU(2)_W$-triplet particles, or by loops involving various new particles.

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[1]General relativity includes an additional mass scale, the cosmological constant, which given current knowledge appears to be independent of the Planck scale. The accelerated expansion of the Universe indicates that the cosmological constant minus the vacuum energy density is tiny (but nonzero). Ignoring issues about fine-tuning (discussed in Section 2), this may be a Standard Model effect because all existing particles contribute to the vacuum energy density.

[2]"Right-handed neutrino" is a potentially confusing name given that left- and right-handed fermions may be interchanged by a charge conjugation operation.

If the neutrino masses are of the Dirac type, then there are right-handed neutrinos which combine with the left-handed ones through Yukawa terms in the Lagrangian:

$$-\lambda_{ij}^{\nu}\bar{\nu}_R^i H L_L^j \tag{2}$$

The Yukawa couplings λ_{ij} must be very small, $\lambda_{ij} \lesssim 10^{-12}$, which implies that the ν_R's cannot be observed directly (unless they happen to have sizable interactions with other new particles [6]). The observation of atmospheric and solar neutrino oscillations has established that at least two of the neutrino masses are nonzero, and therefore there is need for at least two ν_R's. More complicated origins of neutrino masses can also be imagined: for example, a collection of higher-dimensional operators can contribute to both Majorana and Dirac masses [6].

I will refer to the Standard Model plus the graviton and neutrino masses (either Majorana or Dirac) as the SM$\nu\mathcal{G}$, and I will refer to it loosely as the standard model. In the case of only two ν_R's and purely Dirac masses the SM$\nu\mathcal{G}$ has 27 parameters: 19 from the SM, the Planck scale, the cosmological constant, two ν masses, three ν mixing angles, and a ν CP-violating phase; only the latter parameter is experimentally unconstrained for now, and all the parameters from the neutrino sector are dimensionless.

Exercise 1.1: *Count the parameters of the SM$\nu\mathcal{G}$ if the neutrino masses are purely of the Majorana type as in Eq. (1).*

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2 Evidence for physics beyond the standard model

Let us overview the existing experimental and observational information regarding physics beyond the SM$\nu\mathcal{G}$. The only robust piece of information (as of May 2012) comes from various observations of galaxy clusters, the galactic rotation curves, and cosmological data: there is need for at least one new electrically-neutral and stable particle to play the role of dark matter [2]. We do not know what are the spin, mass or interactions of this particle, and in fact we do not know whether there is a single particle or a complicated hidden sector including various kinds of particles. Section 4 reviews some popular theories of dark matter particles.

Currently there are no other convincing experimental results which cannot be explained within the SM$\nu\mathcal{G}$. There are, however, theoretical reasons to expect that there is new physics beyond the SM$\nu\mathcal{G}$. The most often invoked one is the so-called hierarchy problem, discussed in Section 3. Other theoretical problems of the standard model include the following:

• The pattern of measured quark and lepton masses suggests the existence of some underlying mechanism that generates the hierarchies between various fermion masses. Proposed solutions include, for example, fermion masses generated by higher-dimensional operators with coefficients controlled by discrete symmetries, loop-induced masses in the presence of some new particles, or exponential suppressions due to different wave functions along extra dimensions.

• The strong CP problem is the question of why QCD does not lead to large CP violation. The nonobservation of a neutron dipole electric moment implies that the operator $\epsilon_{\mu\nu\lambda\tau}G^{\mu\nu}G^{\lambda\tau}$, with $G^{\mu\nu}$ being the gluon field strength, has a coefficient smaller than 10^{-12}. A nice theoretical explanation for this seemingly unnatural coefficient is the presence of an global $U(1)$ symmetry explicitly broken by a QCD anomaly. This solution implies the existence of an axion field, of spin 0 and very small mass, which although is still allowed by all experimental constraints, is increasingly constrained.

• The $SU(3) \times SU(2) \times U(1)$ charges of the quarks and leptons are strange enough (at least at first sight) to require some explanation in a deeper theory. A beautiful explanation is that the six fields $(q_L, u_R, d_R, l_L, e_R, \nu_R)$ of each generation are exactly the components of a single $SO(10)$ representation. Furthermore, the $SU(3) \times SU(2) \times U(1)$ gauge couplings appear to unify at a scale of about

10^{16} GeV, suggesting an underlying Grand Unified Theory. It is conceivable though that the SM fermions just happen to form full GUT representations without an actual unification gauge group; after in any self-consistent theory the fermion charges are highly restricted by the requirement of gauge anomaly cancellation.

• The asymmetry between matter and antimatter in the observable universe suggests a mechanism for baryogenesis which requires some new sources of CP violation. There is however enough leeway to engineer such a mechanism, for example through the neutrino sector, so that it might not have observable effects in upcoming experiments.

Overall, one should keep in mind that theoretical reasons for new physics are usually based on esthetic prejudice and do not necessarily need to be valid, given the self-consistency of the SM$\nu\mathcal{G}$ up to scales near M_P. Only experimental or observational data that is in disagreement with SM$\nu\mathcal{G}$ (such as that pertaining to dark matter) may provide conclusive evidence for physics beyond the standard model.

Exercise 2.1:   *The $SO(10)$ grand unified gauge group includes an $SU(5) \times U(1)$ subgroup. Assign the SM fermions of one generation to the $10 + \bar{5}$ representation of $SU(5)$.*

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3   Hierarchy problem

The SM$\nu\mathcal{G}$ has two seemingly independent mass scales (ignoring for now the cosmological constant[1]): the electroweak scale v_H and the Planck scale M_P. The ratio of these is tiny, 10^{-17}, which raises the question of whether there is some underlying dynamics that links the two mass scales. Furthermore, quantum effects tend to push the electroweak scale towards the Planck scale, so there is need for some stabilizing mechanism if one does not want to rely on extreme tuning of parameters in the underlying theory. Perturbatively, this is the statement that the squared-mass of the Higgs doublet receives quadratic divergences from loops, primarily involving the top quark but also the Higgs doublet itself and the electroweak bosons.

There are various ideas about how to solve the hierarchy problem, so let us discuss them briefly, in turn. The most explored ones are dynamically broken supersymmetry (Section 3.1) and a warped extra dimension (Section 3.2). However, several other ideas have been proposed:

• *Higgs as a pseudo Nambu-Goldstone boson.*   A global continuous symmetry which is spontaneously broken implies the existence of a massless spin-0 particle, called Nambu-Goldstone boson (NGB). If the global symmetry is also explicitly broken, then the NGB acquires a mass, and is referred to as pseudo NGB. In the limit where the explicit breaking is controlled by a mass parameter much smaller than the VEV responsible for spontaneous breaking, the pseudo NGB is much lighter than other spin-0 particles expected to have mass of the order of the VEV. If the SM is embedded in a theory in which the Higgs doublet arises as a pseudo NGB, then the hierarchy problem is solved. However, it is rather difficult to design such a mechanism without introducing additional elementary scalar fields and thereby reintroducing a hierarchy problem. A well known proposal [7, 8] is to have a new strongly interacting gauge theory acting on some new fermions such that their chiral symmetry is dynamically broken. Some of the ensuing pseudo NGBs (analogues of the pions in QCD) may carry the same quantum numbers as the Higgs doublet. For more recent studies of related theories, see *e.g.* Ref. [9].

• *Composite Higgs boson from top condensation.*   The large mass of the top quark suggests that it is involved in electroweak symmetry breaking. If some new interaction binds a top quark and a top anti-quark within a composite spin-0 field, then at sufficiently strong coupling the composite field acquires a VEV that breaks the electroweak symmetry. The large coupling of the composite field to its constituents (*i.e.*, a $t\bar{t}$ pair) then leads to a large top mass. It turns out, however, that the top is not heavy enough

to accommodate this mechanism unless the scale of the new interaction is many orders of magnitude above the weak scale, which would require fine-tuning. Nevertheless, if there is a new quark with mass of order $1\,\mathrm{TeV}$ which mixes with the top quark, then a bound state of χ with t behaves at low energy exactly as the SM Higgs doublet [10]. To solve the hierarchy problem, it is then necessary to have a new asymptotically-free gauge interaction ('topcolor' [11]) which is spontaneously broken near the scale (in the TeV range) where it becomes strong.

• *Technicolor.* Instead of being broken by the VEV of a Higgs doublet, the electroweak symmetry may be broken by the VEV of a fermion-antifermion pair (usually called a 'fermion bilinear'). This phenomenon actually exists within QCD, where the chiral symmetry breaking is triggered by a $\bar{q}q$ condensate. That breaking however, is three orders of magnitude weaker than that required to fit the W and Z masses. Technicolor is a new gauge theory in which some new gauge interactions breaks the chiral symmetry of some new fermions, called techni-fermions [12]. Given that there is no Higgs doublet or any other elementary scalar field, technicolor solves beautifully the hierarchy problem. However, technicolor faces various phenomenological hurdles.

• *Large extra dimensions (ADD).* If the SM is localized on a 3-dimensional wall embedded in a 3+n-dimensional space and only the graviton propagates in the full space, then the coupling of the graviton to SM particles is suppressed by volume of the extra dimensions [13]. This allows the observed strength of the gravitational interactions to be very small, suppressed by M_P, while the fundamental mass M_* that sets the scale of strongly coupled gravity to be near $1\,\mathrm{TeV}$. As a result, the ADD scenario replaces the hierarchy problem by the question of what makes the volume of the 3+n dimensional space so large in $1/M_*$ units. One could imagine various ways of tackling the latter question. Measurements of gravity are sensitive to the presence of the extra dimensions, and the current upper limit on their radius is around 4×10^{-5} m [14]. The collider implications of this scenario involve emission of KK gravitons which appear as missing transverse momentum, and virtual exchange of KK gravitons which induce nonresonant modifications of various cross sections (the relevant Feynman rules are derived in Ref. [15]).

• *Little Higgs theories.* The 1-loop quadratic divergences in the Higgs self-energy may be cancelled by a set of new particles having the same spin as their SM partners. The divergence due to the top-quark loop is cancelled by the divergence of a top-prime quark loop. The coupling of the top-prime quark to the Higgs boson is given by a dimension-5 operator whose coefficient is linked to the top Yukawa coupling by a symmetry [16]. The extension of this mechanism to more than one loop is not obvious. An interesting version of the Little Higgs mechanism is provided by theories with T parity [17]. These are discussed further in Section 4.

• *Twin Higgs.* Twin Higgs is a mechanism for canceling the 1-loop quadratic divergences similar with Little Higgs in that there is a partner of same spin for each SM particle that has a large contribution to the Higgs self-energy. There is an essential peculiarity of the Twin-Higgs mechanism though: the partners do not carry SM gauge charges [18]. Consequently, there are no easy-to-find signals at the LHC.

Some of the solutions outlined above may also be combined to obtain different mechanisms. For example, the Twin Higgs mechanism may cancel the leading 1-loop divergences, and then the compositeness of the Higgs doublet (for example in a top condensation model) may be natural at a scale in the several TeV range. Given the relatively large number of proposed solutions to the hierarchy problem, and the possibility that many other solutions might be identified in the future, it is difficult to draw firm conclusions based on the requirement of no excessive fine-tuning.

3.1 Dynamically-broken supersymmetry

Supersymmetry is an extension of Lorentz invariance to a superspace that includes an extra dimension with an anticommuting ("fermionic") coordinate. Exact supersymmetry implies that each field has a fermionic and a bosonic component of equal mass, such that loops involving fermions contributing to the squared-mass of the Higgs doublet exactly cancel loops involving bosons.

We know that supersymmetry is not an exact symmetry of nature because, for example, there is no bosonic partner of the electron of mass equal with that of the electron. It is possible, however, that supersymmetry is an exact symmetry of the underlying theory but the ground state of the universe breaks supersymmetry (this is equivalent with a nonzero vacumm energy). If that is the case then only certain terms in the Lagrangian break supersymmetry. These so-called soft susy-breaking terms lead to modified masses for superpartners (and some trilinear interactions between scalar fields). As a result the Higgs quadratic divergences are suppressed at a scale given by the superpartner masses.

To make these statements more precise, let us first discuss the particle content of the super-symmetric theory that includes the SM$\nu\mathcal{G}$, usually called the Minimal Supersymetric Standard Model (MSSM) [19]. For each SM quark, which includes a left-handed and a right-handed component, there are two complex scalar fields called squarks. Likewise, for each SM left- or right-handed lepton there is a complex scalar field called slepton. For each $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge boson there is a spin-1/2 field (a Majorana fermion) generically called gaugino; gluino refers specifically to the $SU(3)_c$ gauge superpartner while wino, zino, and bino (or photino) are the superpartners of the electroweak gauge bosons. The graviton has a spin-3/2 partner called gravitino. Finally, there are two Higgs doublets (in order to have no gauge anomaly), H_u and H_d, and each has a spin-1/2 partner called higgsino.

Supersymmetry ensures that the quadratically divergent part of the top loop contribution to the Higgs mass is cancelled by the loops involving top squarks ("stops"), as shown in Fig. 1. The remnant of this cancellation is of the order of the stop masses times a loop factor, so that the fine-tuning is not worrisome provided the stop masses are not larger than order 1 TeV (more precise statements imply upper bounds of about 500–700 GeV [20]). A light left-handed stop implies that the other scalar from the same $SU(2)_W$ doublet, namely the superpartner of the left-handed bottom quark ("left-handed sbottom"), should also be light [21]. Similarly, loops with Higgs doublets are cancelled by loops with higgsinos, loops with W and Z bosons are cancelled by loops with winos and zinos, and so on for all the particles.

Supersymmetry with soft breaking terms enforces these cancellations of quadratic divergences at any numbers of loops. For practical purposes, corrections involving more than two loops are less relevant for inferring an upper limit on superpartner masses. The gluino contributes at two loops to the Higgs mass, so that one obtains an upper limit of the gluino mass of about 1.5 TeV. This conclusion is more model dependent than the upper limit on the stop masses. For example, in models beyond the MSSM where the gauginos are Dirac particles [22], the two loop contributions are automatically suppressed, so significantly heavier gluinos are natural.

The presence of two Higgs doublets in the MSSM allows a mass term for the Higgs superfields, of the type $\mu H_u H_d$. The parameter μ is a supersymmetric mass, so that its contribution to the mass-squared
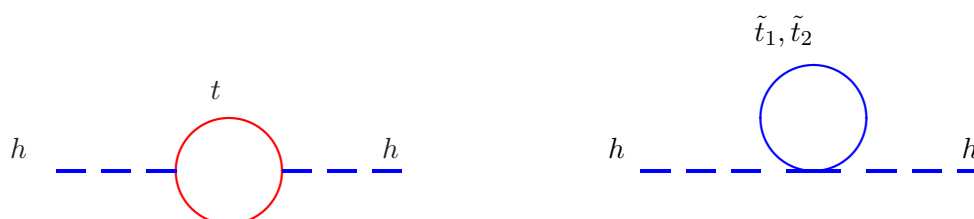

Fig. 1: Quadratic divergences to the self-energy of the Higgs doublet. First diagram is the dominant contribution in the SM, while the second one arises in the MSSM and cancels the quadratic divergence due to the SM top loop.

of each Higgs doublet is $|\mu|^2$ while the higgsino mass is $|\mu|$. This implies that the higgsinos cannot be heavier than the electroweak scale without fine-tuning. The relation between the various contributions to the Higgs square masses can be written as

$$M_Z^2 = -2 \left(M_{H_u}^2 + \delta M_{H_u}^2 + |\mu|^2 \right) , \tag{3}$$

where $\delta M_{H_u}^2$ is the 1-loop correction to the squared mass of H_u:

$$\delta M_{H_u}^2 \approx -3 \frac{\lambda_t^2}{8\pi^2} \left(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 \right) \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right) . \tag{4}$$

where Λ is the scale where the soft-susy terms are generated. Thus, the hierarchy problem requires the MSSM to have soft-susy breaking terms such that the stops have masses near the electroweak scale, and a μ term such that the higgsinos are even lighter. This raises two questions: why are the soft susy masses so much smaller than the Planck scale, and why is the supersymmetric μ term close in size to the soft susy-breaking masses?

There is a nice answer to the first question: a gauge theory with a gauge coupling of order one at the Planck scale may become nonperturbative at a much smaller scale due to the logarithmic running of the gauge coupling. If that type of dynamics leads to a nonzero vacuum energy, then soft susy breaking terms are generated and a large hierarchy between the Planck scale and the scale of supersymmetry breaking may be naturally generated. The gauge theory that breaks supersymmetry is called the dynamically susy-breaking (DSB) sector. Supersymmetry breaking can be transmitted from the DSB sector to the MSSM by gauge interactions (gauge mediation, with $\Lambda \sim O(10^4)\,\text{TeV}$), Planck scale suppressed operators ('supergravity mediation', with $\Lambda \sim O(10^7)\,\text{TeV}$), or other mechanisms.

Given that the μ-term has nothing to do with supersymmetry breaking, there is need for some additional connection between the DSB sector and the generation of the μ term. Although this is highly non-trivial, various realistic models of dynamically broken supersymmetry have been constructed (for example, see Ref. [23] for models of gauge mediation). Nevertheless, these are theories with a rich field content and rather complicated dynamics at various scales between the electroweak and Planck scales. The majority of the phenomenological studies concentrate on the MSSM without being concerned with the exact mechanism of DSB. The phenomenology of the MSSM is discussed in Section 4.

Exercise 3.1: *Check the cancellation of quadratic divergences in Fig. 1.*

3.2 A warped extra dimension: RS1

Consider the existence of an extra spatial dimension of coordinate y, transverse to the usual 3+1 space-time dimensions of coordinates x^μ, such that the line element is given by

$$ds^2 = e^{-2ky} \eta_{\mu\nu} x^\mu x^\nu - y^2 . \tag{5}$$

The 5D Einstein equations need to have a solution given certain boundary conditions. Here k is the AdS$_5$ curvature, and has dimensions of mass. The unit of length along the 4D Minkowski space depends on the position along $y \equiv x^4$. Randall and Sundrum [24] have shown that a slice of AdS$_5$ (space exists only for $0 \leq y \leq L$) is a solution to the 5D Einstein equations provided the vacuum energy $V(y)$ satisfies

$$V(0) = -V(L) = \frac{\Lambda}{k} , \tag{6}$$

where Λ is the 5D cosmological constant. They proposed the following set-up ("RS1"): gravity propagates in the 5D bulk, while all standard model fields are localized at $y = L$. The points $y = 0$ and $y = L$ are usually referred to as the Planck brane and the standard model brane, respectively. It is assumed that
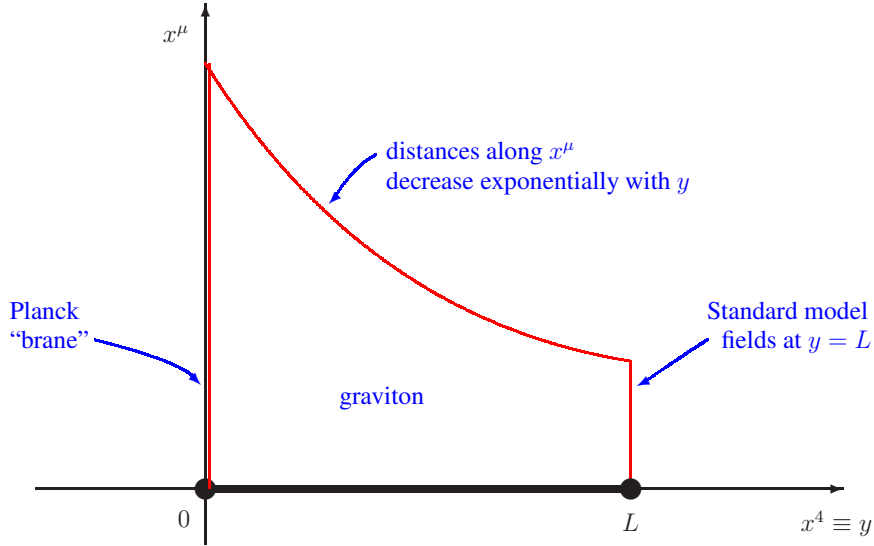
distances along x^μ
decrease exponentially with y

Planck "brane"

Standard model fields at $y = L$

graviton

0 $\qquad L \qquad x^4 \equiv y$

Fig. 2: RS1 model: only the graviton propagates in the warped bulk.

the curvature k is a couple of orders of magnitude below the fundamental 5D scale M_5, where 5D gravity becomes strongly coupled, so that the effective theory has a range of validity. The reduced Planck mass can be derived in terms of M_5 and $k \gg 1/L$:

$$\overline{M}_{\text{Pl}} \approx \sqrt{\frac{M_5^3}{k}} , \qquad (7)$$

and the 5D cosmological constant is given by $\Lambda = -24M_5^3 k^2$.

As a consequence of the metric, any mass parameter m_0 that appears in some terms of the 5D action which are localized at $y = L$ corresponds to a mass m in the effective 4D theory obtained after integrating over y, with

$$m = m_0 \, e^{-kL} . \qquad (8)$$

Thus, if the VEV that breaks the $SU(2)_W \times U(1)_Y$ symmetry in the 5D theory is $v_0 \sim O(k)$, then the corresponding VEV in the 4D effective theory is $v = v_0 e^{-kL}$. Taking $M_5 \approx 10k$ one finds $v \simeq 174\,\text{GeV}$ for $kL \approx 34$. This is a remarkable result: the huge ratio $\overline{M}_{\text{Pl}}/v$ arises from a theory in which there are no large hierarchies between the input mass parameters (M_5, k, $1/L$, v_0).

Exercise 3.2: *Change the Minkowski coordinates in Eq. (5) so that in terms of the new coordinates all distances are given as measured by an observer localized at $y = L$. Show that all input mass parameters are of order the TeV scale in this case, and that the observed weakness of gravity is due to the small wavefunction of the 0-mode graviton on the standard model brane.*

The gravity action [4] in the warped bulk is given by

$$S_{\text{gravity}} = 2M_5^3 \int d^4x \quad \int_0^L dy \, \sqrt{\det G} \, \Big[G^{\alpha\beta} \left(\partial_\beta \Gamma^\delta_{\alpha\delta} - \partial_\delta \Gamma^\delta_{\alpha\beta} \right.$$
$$\left. + \, \Gamma^\eta_{\alpha\delta} \Gamma^\delta_{\eta\beta} - \Gamma^\eta_{\alpha\beta} \Gamma^\delta_{\eta\delta} \right) + 12k^2 \Big] , \qquad (9)$$

where $G^{\alpha\beta}$ is the 5D metric, $\det G$ is the determinant of the metric, and $\Gamma^\delta_{\alpha\beta}$ is the 5D connection:

$$\Gamma^\delta_{\alpha\beta} = \frac{1}{2} G^{\delta\eta} \left(\partial_\alpha G_{\beta\eta} + \partial_\beta G_{\eta\alpha} - \partial_\eta G_{\alpha\beta} \right) . \qquad (10)$$

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Letters from the beginning of the Greek alphabet label the 5D coordinates ($\alpha, \beta, \delta, \eta, ... = 0, 1, 2, 3, 4$), and letters from the middle of the Greek alphabet label the Minkowski coordinates ($\mu, \nu, \rho, \sigma, ... = 0, 1, 2, 3$).

Small gravitational fluctuations are described by expanding the 5D metric about the warped background:

$$G_{\alpha\beta} = \begin{pmatrix} e^{-2ky}\,\eta_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} + \frac{2}{M_5^{3/2}} \begin{pmatrix} e^{-2ky}\,h_{\mu\nu}(x^\rho, y) & h_{\mu4}(x^\rho, y) \\ h_{4\nu}(x^\rho, y) & h_{44}(x^\rho, y) \end{pmatrix} . \tag{11}$$

Here $h_{\mu\nu}$ is the 5D graviton polarized along the Minkowski coordinates, while $h_{\mu4}$ and h_{44} are the polarizations along the extra dimension of the 5D graviton; $h_{\mu4}$ form a 5D spin-1 field called graviphoton, and $h_{\mu4}$ is a 5D spin-0 field called the graviscalar. The 5D graviton $h_{\alpha\beta}$ is a symmetric tensor field, and therefore it has 15 components. However, not all of its components are physical. Imposing a gauge fixing condition that $h_{\alpha\beta}$ is traceless and transverse,

$$h_\alpha^\alpha = 0 \;\;,\;\; \partial^\alpha h_{\alpha\beta} = 0 \;\;, \tag{12}$$

eliminates 10 of the components. The remaining five components are physical degrees of freedom, divided as follows: two in $h_{\mu\nu}$ (a traceless and transverse 4×4 symmetric tensor), two in $h_{\mu4}$ (a massless gauge field with 4-components), and one in h_{44}.

Exercise 3.3: *Keeping only the quadratic terms in $h^{\alpha\beta}$, derive the Lagrangian that describes the propagation of the graviton in the warped background.*

The graviton must have a massless 0-mode, $h_{\mu\nu}^{(0)}$, in order to generate the observed long-range gravitational effects. Hence, the graviton is an even field, *i.e.*, its derivative with respect to y must vanish at $y = 0$ and $y = L$. The KK decomposition for the tensor components of the 5D graviton field is

$$h_{\mu\nu}(x^\rho, y) = \frac{1}{\sqrt{L}} \left[h_{\mu\nu}^{(0)}(x^\rho) + \sum_{j \geq 1} h_{\mu\nu}^{(j)}(x^\rho)\chi_j(y) \right] . \tag{13}$$

The graviton KK modes, $h_{\mu\nu}^{(j)}(x)$, are spin-2 particles in 4D. For $j \geq 1$, they are massive and therefore they have 5 degrees of freedom (corresponding angular momenta ± 2, ± 1 and 0). Two of these degrees of freedom originate in $h_{\mu\nu}(x^\rho, y)$, while the other three are given at each KK level by the graviphoton and graviscalar, which in the unitary gauge disappear from the spectrum. General coordinate invariance requires $h_{\mu4}$ to have odd boundary conditions, and h_{44} to have even boundary conditions [25]. As a result, the 0-mode of h_{44} remains as physical spin-0 particle, which is referred to as the radion. Its properties are analyzed in Ref. [26].

Plugging the KK decomposition (13) into the 5D kinetic terms for $h_{\mu\nu}(x^\rho, y)$ gives the following differential equation for the KK functions [27]:

$$\left[\frac{d}{dy}\left(e^{-4ky}\frac{d}{dy} \right) + m_j^2 e^{-4ky} \right] \chi_j(y) = 0 \;\;, \tag{14}$$

where m_j is the mass of the jth KK mode of spin-2. Solving this equation with the Neumann boundary conditions

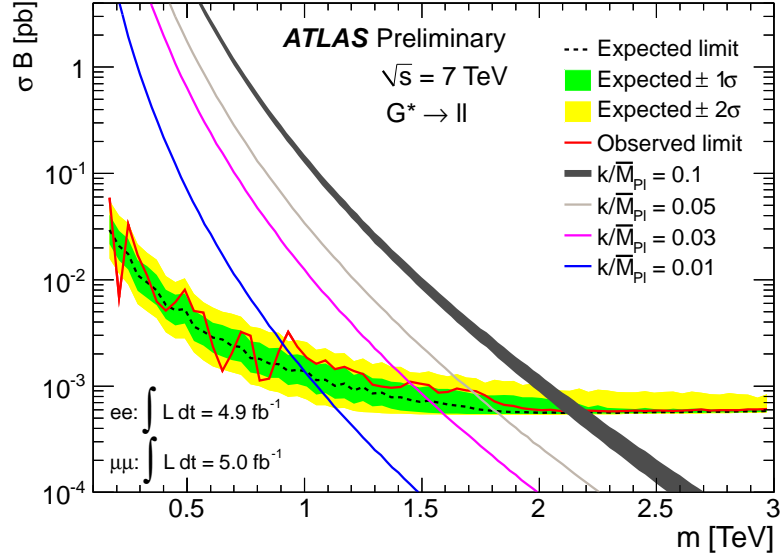$$\frac{d}{dy}\chi_j(y)|_{y=0} = \frac{d}{dy}\chi_j(y)|_{y=L} = 0 \tag{15}$$

Fig. 3: Limits on the cross section times branching fraction for the level-1 graviton of the RS1 model, obtained by the ATLAS Collaboration by searching for a narrow e^+e^- or $\mu^+\mu^-$ resonance. This plot is reproduced from Ref. [28].

leads to KK functions given in terms of the J_2 and Y_2 Bessel functions:

$$\chi_j(y) = \frac{e^{2ky}}{N_j} \left[J_2\left(\frac{m_j}{k}e^{ky}\right) + \alpha_j Y_2\left(\frac{m_j}{k}e^{ky}\right) \right] , \qquad (16)$$

where N_j is a normalization constant, and α_j are constants determined by the boundary conditions (15). The mass of the level-1 KK modes is at the TeV scale, $m_1 \simeq 3.8 k e^{-kL}$, and the higher modes have masses $m_2 \simeq 1.8 m_1$, $m_3 \simeq 2.7 m_1$, $m_4 \simeq 3.5 m_1$, ...

The couplings of the graviton KK modes to standard model fields are identical to those of the graviton 0-mode except for an overall factor of e^{kL}. Therefore, both the masses of the graviton KK modes and the suppression of the couplings are of order the TeV scale. The collider signals of these spin-2 particles are heavy resonances that decay into neutral pairs of standard model particles. The production cross section at the Tevatron or the LHC may be computed from the following couplings to gluons

$$\frac{\sqrt{2}}{\overline{M}_{\rm Pl}} e^{kL} \left(h_\rho^{(j)\rho} \eta_{\mu\nu} - 4 h_{\mu\nu}^{(j)} \right) G_\sigma^\mu G^{\nu\sigma} , \qquad (17)$$

and to quarks

$$\frac{4}{\overline{M}_{\rm Pl}} e^{kL} \left[\left(h_\sigma^{(j)\sigma} \eta_{\mu\nu} - h_{\mu\nu}^{(j)} \right) ((\partial^\nu \bar{q})\gamma^\mu q - \bar{q}\gamma^\mu \partial^\nu q) - m_q h_\sigma^{(j)\sigma} \bar{q}q \right] . \qquad (18)$$

Exercise 3.4: *Compute the parton-level cross sections $gg \to h_{\mu\nu}^{(1)}$ and $q\bar{q} \to h_{\mu\nu}^{(1)}$, in the narrow width approximation, as a function of the curvature k and the $h_{\mu\nu}^{(1)}$ mass.*

The most useful decays of the graviton KK modes are into $\gamma\gamma$, e^+e^- and $\mu^+\mu^-$, due to the small backgrounds. Figure 3 shows the current Tevatron limit on the RS1 model. Figure 4 shows the predicted cross section for $pp \to h_{\mu\nu}^{(j)} \to \ell^+\ell^-$ at the LHC.

Standard model in a warped extra dimension. It is interesting to study what are the effects of the warped metric on the propagation along the extra dimension of fields other than the graviton. Even though this scenario is not motivated by the hierarchy problem, the remainder of this Section is focused on this highly active topic.

Boundary conditions at $y = 0$ and $y = L$ must be specified for each field propagating in the bulk. A gauge field propagating in a warped extra dimension, with Neumann boundary conditions, has a y-independent 0-mode [30]. This flat profile, forced by gauge invariance, is in stark contrast to the exponential profile of the graviton 0-mode. The Kaluza-Klein decomposition for the gauge field is

$$A_\mu(x^\nu, y) = \frac{1}{\sqrt{2L}} \left[A_\mu^{(0)}(x^\nu) + \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) f_j(y) \right]$$

where the KK functions are given by

$$f_j(y) = \frac{e^{ky}}{N_j} \left[J_1\left(\frac{m_j}{k} e^{ky}\right) - \frac{J_0(m_j/k)}{Y_0(m_j/k)} Y_1\left(\frac{m_j}{k} e^{ky}\right) \right]$$

Here J and Y are Bessel functions, N_j is a normalization constant, M_j is the mass of the jth KK mode of spin-1. The level-1 gauge boson has a mass $M_1 = 2.5 k e^{-kL}$, so at the scale associated with the standard model brane, 1.5 times lighter than the level-1 graviton. The higher spin-1 KK modes have masses $M_2 = 2.3 M_1$, $M_3 = 3.6$, $M_3 = 4.8$, ... The couplings of the higher spin-1 KK modes to the fermions localized on the standard model brane are all given by the 4D gauge coupling (*i.e.*, the coupling of the 0-mode gauge boson) times a "volume" factor of $\sqrt{2kL} \approx 8.2$. Therefore, if the standard model gauge bosons propagate in the warped bulk while the quark and leptons are localized at $y = L$, then the spin-1 KK modes are strongly coupled to the fermions. The 4-fermion effective interactions induced by KK exchange are ruled out unless $M_1 > O(20)$ TeV (this is based on the assumption that single boson exchange is a good approximation of the effects induced by the rather strongly coupled KK modes).

If fermions are also propagating in the warped bulk, then their 0-modes have an exponential profile [31]. In addition, the exponential profile may be tuned independently for each fermion flavor, because
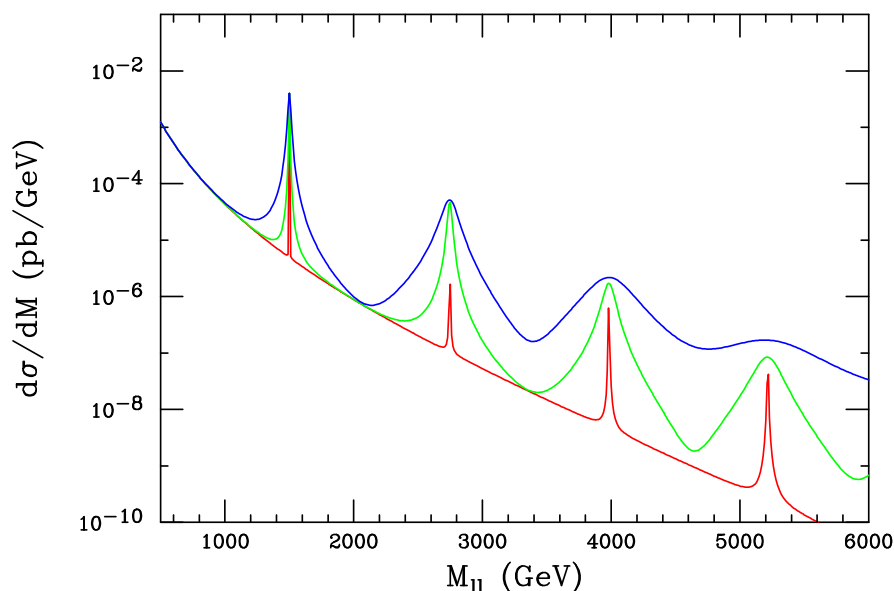

Fig. 4: Theoretical cross section for $pp \to h_{\mu\nu}^{(j)} \to \ell^+\ell^-$ at the 14 TeV LHC [29]. The three curves correspond, from top to bottom, to $k/\overline{M}_{\mathrm{Pl}} = 0.1, 0.05$ and 0.01. The dilepton resonance searches at the 7 TeV LHC already exclude a KK graviton of 1.5 TeV with $k/\overline{M}_{\mathrm{Pl}} > 0.03$, as shown in Fig. 3.

the fermions may have bulk masses. The fermionic part of the 5D Lagrangian takes the following form:

$$\sqrt{g}\left(i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - c_{\psi}k\overline{\psi}\psi\right) , \tag{19}$$

where g is the determinant of the 5D metric. The ensuing y-dependence of the 0-mode is

$$f_{\psi}(y) = f_{\psi}(L)e^{-(2-c_{\psi})k(L-y)/2} . \tag{20}$$

This result may be important for explaining the hierarchies among the standard model fermion masses. The set of lectures [32] is devoted to model building in a warped extra dimension and their holographic interpretation.

Exercise 3.5:   *Show that the fermion bulk mass does not prevent the existence of a massless 0-mode (when the Higgs VEV is neglected).*

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4   Theories of dark matter and their collider signatures

The total mass of dark matter is roughly five times larger than the mass of luminous matter. The most studied and searched for type of dark matter is made of electrically-neutral and stable particles with mass of the order of the electroweak scale. The latter property is motivated by the so-called WIMP miracle: a particle of mass $\sim v_{H}$ and coupling ~ 1 has a relic abundance in agreement with the observed dark matter density.

In order to ensure the stability of the dark matter particle, there is need for a symmetry. The simplest choice is a discrete symmetry such as Z_{2} (a parity). A discrete symmetry may be imposed on many models, leaving many options for the types of WIMPs. There are three popular theories that include a Z_{2} symmetry for dark matter: supersymmetry (see Section 4.1), universal extra dimensions (see Section 4.2) and Little Higgs with T parity [17]. It turns out that there are some deep theoretical connections between these theories: supersymmetry implies the existence of a fermionic extra dimension, Little Higgs with T parity involves a deconstructed extra dimension[3], while UED involves real extra dimensions. At the same time, the properties of the dark matter candidates are completely different in these three theories. For example the spin of the dark matter particle is a fermion in supersymmetry, a boson of spin 1 in minimal UED, and a spin-0 particle in Little Higgs with T parity (as well as in the case of two UEDs).

4.1   Supersymmetric standard model

The Minimal Supersymmetric Standard Model contains many new particles (33) and many new parameters (105), so that it is not surprising that it leads to many interesting signatures. In order to solve the hierarchy problem, the masses of some superpartners must be near the electroweak scale, as discussed in Section 3.1.

Supersymmetry by itself does not guarantee the presence of a dark matter candidate. However, in order to make the MSSM viable, it is necessary to include some discrete symmetry that prevents proton decays. The usual choice (albeit there are many alternatives) for that symmetry is the so called R-parity, under which the SM particles are even while their superpartners are odd. This eliminates the dangerous QQd and QLu operators (when both are present, they lead to fast proton decay). Furthermore, R-parity ensures that the lightest superpartner (LSP) is a stable particle. The mass spectrum of the superpartners depends on the mechanism through which supersymmetry breaking is communicated to the SM fields.

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[3]The construction of 4D theories which are equivalent to 5D theories involving gauge fields and fermions in the bulk is detailed in Ref. [33].
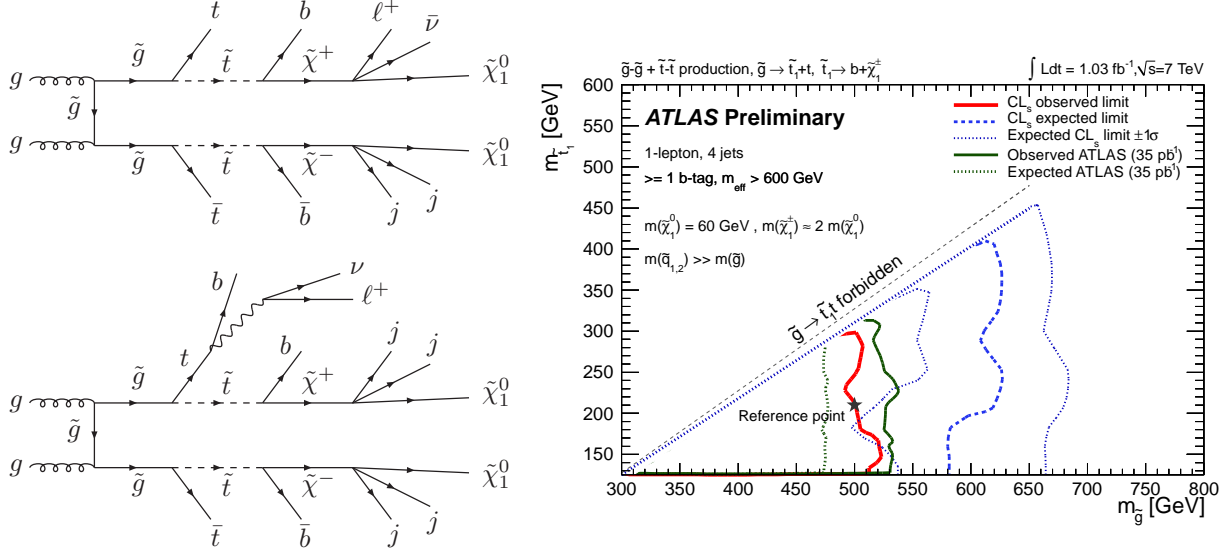
Fig. 5: Left: Typical diagrams for gluino pair production followed by cascade decays involving stops. Right: limits from ATLAS [35] on the stop and gluino masses using the $\ell + b$ jets $+\not{E}_T$.

In many instances the lightest superpartner is the bino, but there are well studied models where it is the gravitino. Both of these particles are electrically-neutral and represent viable dark matter candidates.

R-parity implies that superpartners can be produced only in pairs and that they undergo a series of decays that end with the LSP. The gluino is a color-octet fermion, so that QCD requires an interaction of a gluon to two gluinos with a coupling set by the strong gauge coupling $g_s = \sqrt{4\pi\alpha_s}$. As a result, at hadron colliders the production cross section is rather large for gluino pair production. Thus, the most common signatures involve the cascade decays of two gluinos, which necessarily involve a number of jets and either missing energy due to the LSPs escaping the detector or charged stable particles if their lifetime is long enough to have displaced vertices. The squarks are color-triplet scalars, so that there are interactions of two squarks with one or two gluons, but the cross section for pair production of squarks is an order of magnitude smaller than that of gluinos of comparable mass.

A popular assumption about the mass spectrum is that the gauginos have a unified mass at the GUT scale, and also the squarks and sleptons have the same mass at the GUT scale, which leads to a certain mass spectrum at the TeV scale. The typical signature is jets plus missing E_T plus leptons [19]. Given the nonobservation of this signature implies that the superpartner masses are above the TeV scale, which makes this scenario less appealing.

A less constrained scenario is that where the third generation squarks are the only light squarks (see *e.g.* Ref. [34]). Their decays into a quark and the LSP then lead to the final states as those shown in Fig. 5. Note that the production mechanism in Fig. 5 relies on not too heavy gluinos. A more convincing test of MSSM is direct $\tilde{t}\tilde{t}^\dagger$ production. However, this has small rate, large background, and there is model dependence in the final state: for example, $t\bar{t} + \not{E}_T$ if the LSP is the bino, or $t\bar{t}\tau^+\tau^-\tau^+\tau^- + \not{E}_T$ in gauge-mediation where the LSP is the gravitino, as shown in Fig. 6.

Another important test of the MSSM is direct production of higgsinos, because their mass is set by the electroweak scale, as explained in Section 3.1. To be more precise, the physical states are not just higgsinos, but rather admixtures of higgsinos and electroweak gauginos called charginos (charge ± 1) or neutralinos (electrically neutral). An off-shell W boson can produce a chargino and a neutralino, leading for example to relatively clean $3\ell + \not{E}_T$ signatures, but the rate for this process is an order of magnitude smaller than squark pair production.
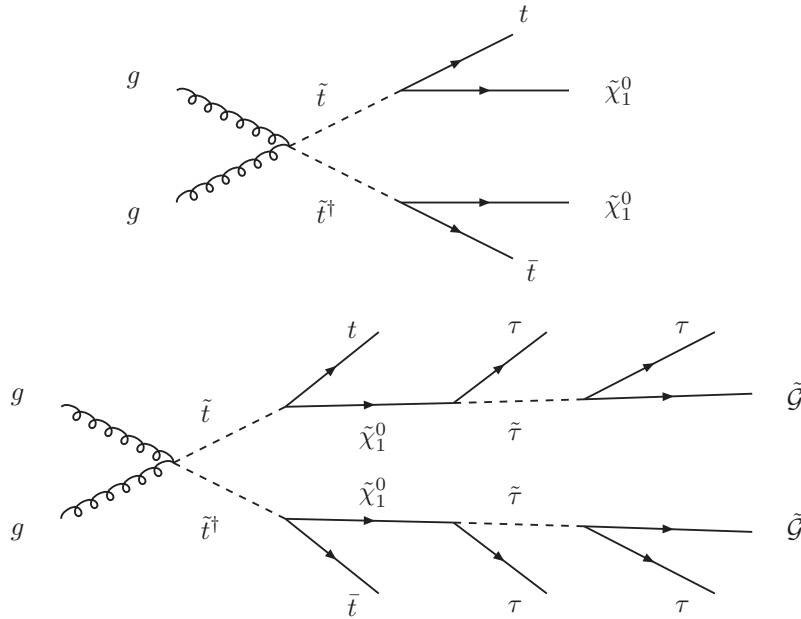
Fig. 6: Direct production of a stop pair leading to the $t\bar{t} + E\!\!\!/_T$ or $t\bar{t} + 4\tau + E\!\!\!/_T$ final states.

Exercise 4.1a: *Draw Feynman diagrams for stop pair production followed by cascade decays involving the chargino ($\chi^{\pm}$) decay to a $W^{\pm}$ boson and a neutralino (χ_1^0). Propose search strategies of these processes at the LHC.*

Exercise 4.1b: *Find the final states arising from chargino pair production (through an off-shell Z or photon), due to the chargino interactions with a W and a neutralino, or with a lepton and a slepton.*

4.2 Universal extra dimensions

Any particle propagating through extra dimensions, whether compactified on a circle or an interval, would appear in experiments as a tower of massive particles in 3 spatial dimensions. The presence of these KK modes can be easily understood based on the usual particle-in-a-box problems in quantum mechanics: given that space along the extra dimensions is compact, the energy states are quantized. The kinetic energy due to motion along the extra dimensions manifests itself as mass in the usual 3 spatial dimensions. For an interval of length L, the mass of the lightest KK modes is $(\pi/L)\hbar/c$ (the natural unit system, $\hbar = c = 1$, is used in what follows). This mass is the compactification scale, and its inverse $R \equiv L/\pi$ is the 'radius' of the extra dimension.

Thus, any extra-dimensional field theory is equivalent to a 4-dimensional one that includes a series of heavy particles. The spectrum and interactions of these KK particles depend on the boundary conditions and metric. Circle compactification would not allow any particle discovered so far to propagate along the extra dimension. The reason is that any gauge field, such as the photon, would include a spin-0 partner of equal mass and couplings as the spin-1 particle. Furthermore, any fermion that propagates along the extra dimension would be a vectorlike fermion: its left- and right-handed components would have the same gauge charges, which is not true for any of the elementary fermions discovered so far. However, if the compactification is on an interval, then the unwanted vectorlike partners of the observed fermions and the spin-0 partners of the gauge fields may be eliminated by the boundary conditions at the end of the interval.

If all bosons propagate in extra dimensions while the fermions are localized at the end points of

an interval [36, 37], the KK modes of the standard model gauge bosons appear as s-channel resonances at the LHC so that one can set limits on $1/R$ of several TeV.

Universal extra dimensions (UED) are arguably the simplest kind of extra dimensions: all particles propagate along some flat compact extra dimensions. The remarkable feature of UED is that a remnant of translational invariance along the extra dimensions is preserved such that a single KK mode cannot couple at tree-level to zero modes [38]. As a result, the limits are relaxed by an order of magnitude compared to the extra dimensions accessible only to bosons. Furthermore, UED lead to dramatically different phenomenological implications. The lightest KK particle is a dark matter candidate, and the collider signals include cascade decays involving leptons, jets and missing energy, as well as narrow resonances.

Field theories in extra dimensions are strongly coupled in the ultraviolet, so that their study could shed light on nonperturbative phenomena, with possible applications to dynamical electroweak symmetry breaking and compositeness. A useful review of strongly coupled theories and their relation to extra dimensions is given in Ref. [12]. A thorough presentation of the phenomenology of various extra dimensional models can be found in the TASI lectures of G. Kribs [39]. The implications of universal extra dimensions for dark matter, which continue to be analyzed by many groups, are reviewed in Ref. [40].

4.2.1 Field theory on a flat compact dimension

Before discussing the phenomenology of extra dimensions, it is necessary to study the general features of quantum field theory in a flat extra dimension. The cases of spin 0, 1/2 and 1 are analyzed in turn.

Scalar field on the interval. Let us consider a five-dimensional (5D) spacetime: four spacetime dimensions of coordinates x^μ, $\mu = 0, 1, 2, 3$, form the usual Minkowski spacetime, and one transverse spatial dimension of coordinate x^4 is flat and compact, with $0 \leq x^4 \leq L$. Thus the extra dimension is an interval (see Fig. 7), and the boundary conditions at its end points determine the spectrum of KK modes.

Free scalar fields, $\Phi(x^\mu, x^4)$, are described by the following action:

$$S_\Phi = \int d^4x \int_0^L dx^4 \left(\partial_\alpha \Phi^\dagger \partial^\alpha \Phi - M_0^2 \Phi^\dagger \Phi \right) . \tag{1}$$

The parameter M_0 is the 5D mass of Φ. We use letters from the beginning of the Greek alphabet to label the 5D coordinates $\alpha, \beta, ... = 0, 1, 2, 3, 4$, and letters from the middle of the Greek alphabet to label the Minkowski coordinates $\mu, \nu, ... = 0, 1, 2, 3$. Given that the action is dimensionless, and that the coordinates have mass dimension -1, the 5D bosons have mass dimension $+3/2$.

Under a variation of the field, $\delta\Phi(x^\mu, x^4)$, the variation of the action is given by

$$\delta S_\Phi = \delta S_\Phi^v + \delta S_\Phi^s , \tag{2}$$

where the first term is a 'volume' integral,

$$\delta S_\Phi^v = - \int d^4x \int_0^L dx^4 \left(\partial^\alpha \partial_\alpha \Phi^\dagger + M_0^2 \Phi^\dagger \right) \delta\Phi , \tag{3}$$

Fig. 7: The extra dimension of coordinate x^4 extends from $x^4 = 0$ to $x^4 = L$, and is transverse to the usual three spatial dimensions.

and the second term is a 'surface' integral,

$$\delta S_\Phi^s \;=\; \int d^4x \left(\partial_4\Phi^\dagger \delta\Phi \Big|_{x^4=L} - \partial_4\Phi^\dagger \delta\Phi \Big|_{x^4=0} \right) .$$

(4)

Here we have assumed as usual that the field vanishes at $x^\mu \to \pm\infty$. Given that the action has to be stationary with respect to *any* variation of the field, the volume and surface terms must vanish independently. Requiring $\delta S_\Phi^v = 0$ implies that Φ is a solution to the 5D Klein-Gordon equation,

$$\left(\partial^\mu \partial_\mu - \partial_4^2 + M_0^2 \right) \Phi = 0 ,$$

(5)

while $\delta S_\Phi^s = 0$ forces the boundary conditions that can be imposed on Φ to obey

$$(\partial_4\Phi^\dagger)\delta\Phi \Big|_{x^4=L} = (\partial_4\Phi^\dagger)\delta\Phi \Big|_{x^4=0} .$$

(6)

Given that the values of $\delta\Phi(x^\mu, x^4)$ at $x^4 = 0$ and $x^4 = L$ are in general not correlated (unless the two points are identified, which would not allow chiral fermions in the 4D theory), and Eq. (6) must be valid for any $\delta\Phi$, both the left- and right-handed sides of Eq. (6) must vanish. Therefore,

$$\partial_4\Phi|_{x^4=0} = 0 \quad \text{or} \quad \Phi(x^\mu, 0) = 0$$

(7)

and

$$\partial_4\Phi|_{x^4=L} = 0 \quad \text{or} \quad \Phi(x^\mu, L) = 0 .$$

(8)

We now solve the 5D Klein-Gordon equation,

$$\left(\partial^\mu \partial_\mu - \partial_4^2 + M_0^2 \right) \Phi = 0 ,$$

(9)

subject to the boundary conditions (7) and (8). Since the boundary conditions are independent of x^μ, then Φ can be decomposed in Fourier modes as follows:

$$\Phi(x^\mu, x^4) = \sum_j \Phi^{(j)}(x^\mu) f^j(x^4) .$$

(10)

The 4D scalar fields $\Phi^{(j)}$ ('KK modes' or excitations), satisfy

$$\left(\partial^\mu \partial_\mu + M_0^2 + M_j^2 \right) \Phi^{(j)}(x^\mu) = 0 ,$$

(11)

where M_j^2 is a positive eigenvalue. The f^j functions are solutions to the one-dimensional equation,

$$\left(\partial_4^2 + M_j^2 \right) f^j(x^4) = 0 .$$

(12)

A general solution to the above equation is

$$f^j(x^4) = C_+ e^{ijx^4/R} + C_- e^{-ijx^4/R} ,$$

(13)

where $C_\pm$ are complex coefficients, and j is a real number such that

$$M_j = \frac{j}{R} ,$$

(14)

and we defined the 'compactification radius'

$$R \equiv \frac{L}{\pi} .$$

(15)

The boundary conditions (7) and (8) impose a relation between the two coefficients, $C_- = \pm C_+$, and also restrict the values of j: $e^{4ij\pi} = 1$. Furthermore, the normalization condition,

$$\int_0^L dx^4 \left[f^j(x^4) \right]^* f^{j'}(x^4) = \delta_{jj'} \ , \tag{16}$$

determines the last coefficient up to a phase factor which we choose to be one. Explicitly, the solutions to Eq. (12) can be written as

$$f_0^j(x^4) = \frac{1}{\sqrt{L(1 + \delta_{j,0})}} \cos\left(\frac{jx^4}{R} \right) \tag{17}$$

for $\partial_4 \Phi|_{x^4=0} = \partial_4 \Phi|_{x^4=L} = 0$ (Neumann boundary conditions),

$$f_1^j(x^4) = \frac{1}{\sqrt{L}} \sin\left(\frac{jx^4}{R} \right) \tag{18}$$

for $\Phi(x^\mu, 0) = \Phi(x^\mu, L) = 0$ (Dirichlet boundary conditions),

$$f_2^j(x^4) = \frac{1}{\sqrt{L}} \sin\left(\frac{(j - 1/2)x^4}{R} \right) \tag{19}$$

for $\Phi(x^\mu, 0) = \partial_4 \Phi|_{x^4=L} = 0$ (mixed boundary conditions),

$$f_3^j(x^4) = \frac{1}{\sqrt{L}} \cos\left(\frac{(j - 1/2)x^4}{R} \right) \tag{20}$$

for $\partial_4 \Phi|_{x^4=0} = \Phi(x^\mu, L) = 0$ (mixed boundary conditions),

with j an integer called 'KK number'.

The functions f_n^j form a complete orthonormal set on the interval if

$$\sum_j \left[f_n^j(x^4) \right]^* f_n^{j)}(x'^4) = \delta(x'^4 - x^4) \ . \tag{21}$$

The allowed values for j must be chosen such that the above completeness condition is satisfied. It is straightforward to check that $j \geq 0$ for $n = 0$, and $j \geq 1$ for $n = 1, 2, 3$. For $n = 0$ there is a state ($j = 0$) of zero momentum ('zero mode') along the compact dimension.

Exercise 4.2: *Integrate the action (1) over x^4 and show that the 4D particles $\Phi^{(j)}(x^\mu)$ have masses*

$$M^{(j)} = \sqrt{M_0^2 + \frac{j^2}{R^2}} \ , \tag{22}$$

if the boundary conditions are of the type $n = 0$ or 1, and

$$M^{(j)} = \sqrt{M_0^2 + \frac{(j - 1/2)^2}{R^2}} \ , \tag{23}$$

if $n = 2$ or 3.

In what follows we will concentrate on the KK functions f_0 and f_1, which are usually referred to as even and odd, respectively; the corresponding boundary conditions represent the so called S^1/Z_2 orbifold.

Fermions on the interval: chiral boundary conditions. We now turn to free spin-1/2 fields in five dimensions. The Clifford algebra is generated by five anti-commuting matrices: Γ^α, $\alpha = 0, 1, 2, 3, 4$. The minimal dimensionality of these matrices is 4×4. The Γ^α matrices can be used to construct a spinor representation of the $SO(1,4)$ Lorentz group, with the generators explicitly given by

$$\frac{\Sigma^{\alpha\beta}}{2} = \frac{i}{4} [\Gamma^\alpha, \Gamma^\beta] \ . \tag{24}$$

The spin-1/2 fermions in five dimensions have four components. In terms of the usual γ^μ matrices used in 4D field theory, one may take $\Gamma^\mu = \gamma^\mu$ and $\Gamma^4 = i\gamma_5$. The fact that $\Gamma^4 \propto \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3$ implies that the $SO(1,4)$ Lorentz group has a single spin-1/2 representation, and thus the 5D fermions are vectorlike.

Upon compactification of x^4, the $SO(1,3)$ Lorentz symmetry generated by $\Sigma^{\mu\nu}/2$, $\mu, \nu = 0, 1, 2, 3$, remains unbroken. There are two chiralities under $SO(1,3)$, labeled as usual by L and R. These are projected by

$$P_{L,R} = \frac{1}{2} \left(1 \pm i\Gamma^4\right) \ . \tag{25}$$

A 5D fermion, Ψ, decomposes into two fermions of definite chirality under $SO(1,3)$:

$$\Psi(x^\mu, x^4) = \Psi_L(x^\mu, x^4) + \Psi_R(x^\mu, x^4) \ , \tag{26}$$

where

$$\Psi_{L,R} \equiv P_{L,R} \Psi \ . \tag{27}$$

As in Section 2, we consider the compactification on an interval: $0 \leq x^4 \leq L$. The action for a free 5D field of spin 1/2 and mass zero is

$$S_\Psi = \int d^4x \int_0^L dx^4 \frac{i}{2} \left[\overline{\Psi} \Gamma^\alpha \partial_\alpha \Psi - \left(\partial_\alpha \overline{\Psi}\right) \Gamma^\alpha \Psi \right] \ . \tag{28}$$

Note that the 5D fermions have mass dimension +2. Under an arbitrary variation of the field, $\delta\Psi(x^\mu, x^4)$, the action has to be stationary both inside the interval and on its boundary:

$$\delta S_\Psi^v = -\int d^4x \int_0^L dx^4 i \left(\partial_\alpha \overline{\Psi}\right) \Gamma^\alpha \delta\Psi = 0 \ ,$$

$$\delta S_\Psi^s = \frac{i}{2} \int d^4x \int_0^L dx^4 \left(\overline{\Psi} \Gamma^4 \delta\Psi \big|_{x^4=L} - \overline{\Psi} \Gamma^4 \delta\Psi \big|_{x^4=0} \right)$$
$$= 0 \ . \tag{29}$$

The first equation implies that Ψ is a solution to the 5D Weyl equation, which can be decomposed into two equations:

$$\Gamma^\mu \partial_\mu \Psi_L = -\Gamma^4 \partial_4 \Psi_R \ ,$$
$$\Gamma^\mu \partial_\mu \Psi_R = -\Gamma^4 \partial_4 \Psi_L \ . \tag{30}$$

The second equation (29) restricts the values of Ψ on the boundary.

In the case of a fermion whose zero-mode is left-handed, the boundary conditions are as follows:

$$\partial_4 \Psi_L(x^\mu, 0) = \partial_4 \Psi_L(x^\mu, L) = 0 \ ,$$

$$\Psi_R(x^\mu, 0) = \Psi_R(x^\mu, L) = 0 \ . \tag{31}$$

The ensuing KK decomposition is given by

$$\Psi = \frac{1}{\sqrt{L}} \left\{ \Psi_L^{(0)}(x^\mu) + \sqrt{2} \sum_{j \geq 1} \left[\Psi_L^{(j)}(x^\mu) \cos\left(\frac{jx^4}{R}\right) \right. \right.$$

$$\left. \left. + \Psi_R^{(j)}(x^\mu) \sin\left(\frac{jx^4}{R}\right) \right] \right\} . \tag{32}$$

All fermion KK modes for $j \geq 1$ pair up to form vectorlike fermions of Dirac masses $M^{(j)}$ as given in Eq. (22). In the case of a fermion whose zero-mode is right-handed, the above equations apply with left- and right-handed labels interchanged. The conclusion is that the boundary conditions for the left- and right-handed fermions are forced by the stationary of the action to eliminate the zero mode for one of the chiralities. This is true for the interval compactification discussed here, while the compactification on a circle would preserve both the left- and right-handed zero modes.

Exercise 4.3: *Write down an Yukawa interaction between a bulk fermion and a bulk scalar, and show that the 4D Yukawa coupling λ (obtained after integration over x^4) is given in terms of the 5D Yukawa coupling λ_5 by*

$$\lambda = \frac{\hat{\lambda}}{\sqrt{L}} . \tag{33}$$

Gauge fields on the interval. A 5D gauge boson has five components: $A_\mu(x^\nu, x^4)$, $\mu, \nu = 0, 1, 2, 3$, and $A_4(x^\nu, x^4)$ which corresponds to the polarization along the extra dimension. From the point of view of the 4D theory, A_4 is a tower of spinless KK modes.

Exercise 4.4: *Write down the action for a 5D gauge boson, and then imposing the stationarity of the action under an arbitrary field variation, derive the field equations.*

The boundary conditions consistent with gauge invariance are given by

$$\partial_4 A_\mu(x^\nu, 0) = \partial_4 A_\mu(x^\nu, L) = 0 ,$$

$$A_4(x, 0) = A_4(x, L) = 0 . \tag{34}$$

Solving the field equations with these boundary conditions yields the following KK expansions:

$$A_\mu = \frac{1}{\sqrt{L}} \left[A_\mu^{(0)}(x^\nu) + \sqrt{2} \sum_{j \geq 1} A_\mu^{(j)}(x^\nu) \cos\left(\frac{jx^4}{R}\right) \right] ,$$

$$A_4 = \sqrt{\frac{2}{L}} \sum_{j \geq 1} A_4^{(j)}(x^\nu) \sin\left(\frac{jx^4}{R}\right) . \tag{35}$$

The zero-mode $A_\mu^{(0)}(x^\nu)$ is the massless gauge boson associated with the 4D gauge transformations. Note that A_4 does not have a zero-mode. In the unitary gauge, the $A_4^{(j)}(x^\nu)$ KK modes are the longitudinal components of the heavy spin-1 KK modes $A_\mu^{(j)}(x^\nu)$.
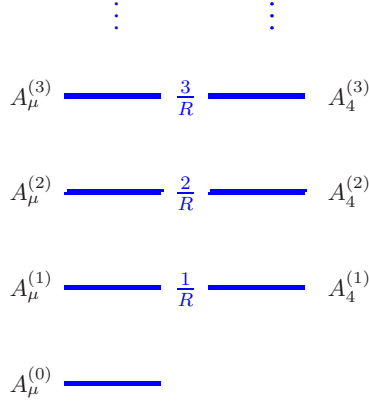
$$
\begin{array}{ccc}
\vdots & & \vdots \\
A_\mu^{(3)} \quad \underline{\quad\quad} & \tfrac{3}{R} & \underline{\quad\quad} \quad A_4^{(3)} \\
\\
A_\mu^{(2)} \quad \underline{\quad\quad} & \tfrac{2}{R} & \underline{\quad\quad} \quad A_4^{(2)} \\
\\
A_\mu^{(1)} \quad \underline{\quad\quad} & \tfrac{1}{R} & \underline{\quad\quad} \quad A_4^{(1)} \\
\\
A_\mu^{(0)} \quad \underline{\quad\quad} & &
\end{array}
$$

Fig. 8: The spectrum of KK modes for the gauge boson associated with an unbroken gauge symmetry.

Assuming that the gluon is a 5D field and that the quarks are localized at $x^4 = 0$, the terms of the 4D Lagrangian describing the interactions between gluon KK modes and quarks are given by

$$
\begin{aligned}
\mathcal{L}_{4D} &= \int_0^L dx^4 \, \hat{g} G_\mu^a(x^\nu, x^4) \left[\delta\left(x^4\right) \bar{q}(x^\nu)\gamma^\mu T^a q(x^\nu) \right] \\
\\
&= g_s \left(G_\mu^{(0)a} + \sqrt{2} \sum_{j \geq 1} G_\mu^{(j)a}(x) \right) \bar{q}\gamma^\mu T^a q \qquad (36)
\end{aligned}
$$

From the above equation follows that the 4D gauge coupling g_s is given in terms of the 5D gauge coupling $\hat{g}$ by $g_s = \hat{g}/\sqrt{L}$, and that the coupling of any gluon KK mode to quarks is larger than the QCD coupling by a factor of $\sqrt{2}$. This factor is a consequence of the normalization of the KK functions, which in turn follows from the canonical normalization of the kinetic terms.

If the gauge symmetry is broken by the vacuum expectation value, v, of a bulk scalar field Φ with Neumann boundary conditions, then the KK modes of the CP-odd components of Φ mix with the corresponding $A_4^{(j)}$. At each KK level one linear combination becomes the longitudinal degree of freedom of $A_\mu^{(j)}$ and the orthogonal one appears as a spin-0 particle. The masses of both $A_\mu^{(j)}$ and of the spin-0 particles are given by

$$
M_A^{(j)} = \sqrt{g^2 v^2 + \frac{j^2}{R^2}} \ . \qquad (37)
$$

where g is the 4D gauge coupling.

4.2.2 One universal extra dimension

Being equipped with the basics of field theory in five dimensions, we can now discuss the case where all standard model particles propagate along one flat extra dimension compactified on an interval, *i.e.,* one universal extra dimension.

Ignoring electroweak-symmetry breaking effects, the tree-level spectrum consists of equally spaced KK levels (of mass j/R), and on each level the KK modes for all standard model particles are degenerate. Each standard model chiral fermion has a tower of vectorlike modes. The KK spectrum of the $(t, b)_L$ doublet and t_R is illustrated in Fig. 9.

To understand the effects of electroweak symmetry breaking on the KK fermion spectrum, let us analyze the case of the top quark (the same applies to the other standard model fermions, except that

electroweak symmetry breaking effects are suppressed by their small Yukawa couplings). Let us denote the bulk fermion whose zero mode is t_L (which is part of a weak doublet $\mathcal{Q}_3$) by $\mathcal{Q}_t(x^\mu, x^4)$, and the bulk fermion whose zero mode is t_R (weak singlet) by $\mathcal{U}_3(x^\mu, x^4)$. The terms of the 4D Lagrangian responsible for top KK masses come from the kinetic terms along the extra dimension and from the Yukawa couplings to the Higgs doublet:

$$\mathcal{L}_{4D} = \int_0^L dx^4 \left[\overline{\mathcal{Q}}_t i\gamma^4 (-\partial_4) \mathcal{Q}_t + \overline{\mathcal{U}}_3 i\gamma^4 (-\partial_4) \mathcal{U}_3 + \left(-\hat{\lambda}_t H \overline{\mathcal{Q}} \mathcal{U}_3 + \text{H.c.} \right) \right] \tag{38}$$

The bulk Higgs doublet, H, which is an even field, has a negative-squared 5D mass; this leads to a vacuum expectation value $v \simeq 174\,\text{GeV}$ only for the zero mode $H^{(0)}$. Inserting now the KK decompositions for $\mathcal{Q}_t$ and $\mathcal{U}_3$, which are analogous to Eq. (32), we obtain the following terms responsible for level-j top masses:

$$\frac{-2}{L} \int_0^L dx^4 \left\{ \overline{\mathcal{Q}}_{tR}^{(j)} \mathcal{Q}_{tL}^{(j)} \sin\frac{jx^4}{R} \partial_4 \cos\frac{jx^4}{R} + \overline{\mathcal{U}}_{3R}^{(1)} \mathcal{U}_{3L}^{(j)} \cos\frac{jx^4}{R} \partial_4 \sin\frac{jx^4}{R} \right.$$

$$\left. + \lambda_t v \left[\overline{\mathcal{Q}}_{tR}^{(j)} \mathcal{U}_{3L}^{(j)} \sin^2\left(\frac{jx^4}{R}\right) + \overline{\mathcal{U}}_{3R}^{(j)} \mathcal{Q}_{tL}^{(j)} \cos^2\left(\frac{jx^4}{R}\right) \right] \right\} + \text{H.c.} \tag{39}$$

where $\lambda_t = \hat{\lambda}_t / \sqrt{L} \approx 1$ is the standard model top Yukawa coupling. From the above equation it is clear that electroweak symmetry breaking leads to mixing between the towers of vectorlike fermions that have t_L and t_R as zero modes. After integration over x^4 we get the mass terms for the top quark KK modes at level j:

$$\left(\overline{\mathcal{Q}}_{tR}^{(j)}, \overline{\mathcal{U}}_{3R}^{(j)} \right) \begin{pmatrix} -\frac{j}{R} & \lambda_t v \\ \lambda_t v & \frac{j}{R} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_{tL}^{(j)} \\ \mathcal{U}_{3L}^{(j)} \end{pmatrix}. \tag{40}$$

Due to the minus sign in the 11 element, the two eigenvalues are equal at tree level:

$$M_{t_1}^{(j)} = M_{t_2}^{(j)} = \sqrt{\frac{j^2}{R^2} + m_t^2} \,. \tag{41}$$

The mass degeneracy among the level-j modes of various standard model particles is further lifted by loop corrections [42]. An important property of any type of interaction in a 5D theory is that it grows with the energy, such that it becomes nonperturbative at a certain energy scale Λ. In the equivalent 4D description that includes a tower of KK modes, the number of KK modes grows with the energy
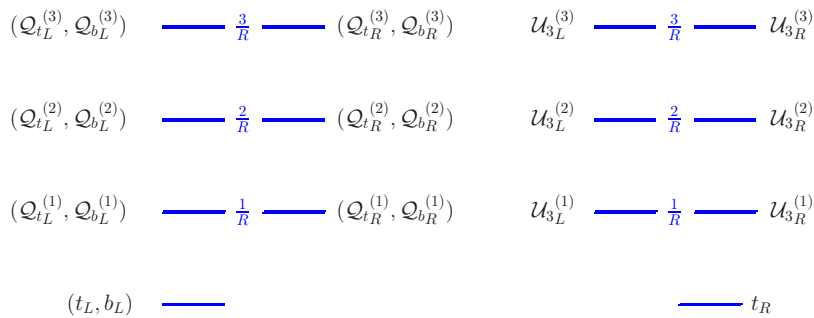

Fig. 9: The tree-level spectrum of top-quark KK modes, ignoring electroweak symmetry breaking.

scale such that the loop corrections are divergent; they need to be cutoff at a scale Λ, which means that the tower of modes is truncated. In practice, the QCD interactions become nonperturbative in the UV at scales roughly two orders of magnitude above the compactification scale. Hence, one can study perturbatively the effective theory below the scale Λ, but higher-dimensional operators suppressed by that scale are likely to be generated by physics at scales above Λ. Working within this effective theory, the loop corrections to KK masses are logarithmically dependent on Λ. For fermions, the leading 1-loop corrections may be obtained by substituting

$$\frac{j}{R} \to \frac{j}{R} \left[\sum_{i=1}^{3} 9 C_i \alpha_i - (3 - 2C_2) \frac{\lambda_f^2}{4\pi} \right] \frac{1}{4\pi} \ln(\Lambda R/j) \tag{42}$$

in the tree-level mass formulas. Here α_i for $i = 1, 2, 3$ are the $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$ coupling constants, $C_1 = y_f^2$ for fermions of hypercharge y_f (using the normalization where y_f is the electric charge for weak singlets), $C_2 = 3/4$ for $SU(2)_W$ doublets and 0 for singlets, $C_3 = 4/3$ for quarks and 0 for leptons, and λ_f is the fermion Yukawa coupling. Note that the $SU(2)_W$ loop corrections to the top KK masses split the diagonal elements in Eq. (40), and therefore $t_1^{(j)}$ and $t_2^{(j)}$ (which are the physical Dirac fermions representing the top modes at each level) end up with different masses.

For gauge bosons, the leading loop corrections are given by the following substitution in the tree-level masses:

$$\frac{j^2}{R^2} \to \frac{j^2}{R^2} C_i' \frac{\alpha_i}{4\pi} \ln(\Lambda R/j) , \tag{43}$$

where C_i' equals to 23 for the gluon, to 15 for the $SU(2)_W$ bosons, and to $-1/3$ for the hypercharge boson B_μ. The different loop contributions to the W_μ^3 and B_μ KK masses have a dramatic effect on their mixing due to electroweak symmetry breaking: the mixing vanishes in the limit where $1/R \gg v$. For this reason, the KK modes of the photon are labelled in the literature by either $\gamma^{(j)}$ or $B^{(j)}$, and the Z modes are labelled by either $Z^{(j)}$ or $W^{3(j)}$.

Exercise 4.5: *Compute the masses of $\gamma^{(j)}$ and $Z^{(j)}$ as a function of $1/R$, v, and Λ.*

Exercise 4.6: *Write down a Higgs mass term localized at both $x^4 = 0$ and $x^4 = L$, and determine its effects on the KK Higgs masses.*

It is often assumed that higher-dimensional operators (in particular, kinetic terms localized on the boundary) and localized Higgs mass terms may be ignored at the scale Λ [42]. The lightest KK particle is then the first KK mode of the photon, and the heaviest particles at each level are the KK modes of the gluon and quarks. The mass spectrum of all level-1 particles is shown in Fig. 10. If the unspecified UV completion of this low-energy effective theory gives rise to operators localized at the ends of the interval, then the KK spectrum may change; this possibility is not considered in what follows.

Momentum conservation along the extra dimension is broken by the boundary conditions, but a remnant of it is left intact. This is reflected in a selection rule for the KK-numbers of the particles participating in any interaction. A vertex with particles of KK numbers $j_1, \ldots, j_p$ exists at tree level only if $j_1 \pm \ldots \pm j_p = 0$ for a certain choice of the $\pm$ signs. This selection rule has important phenomenological implications. First, it is not possible to produce only one KK 1-mode at colliders. Second, tree-level exchange of KK modes does not contribute to currently measurable quantities. Therefore, the corrections to electroweak observables are loop suppressed, and the limit on $1/R$ from electroweak measurements is rather weak, of the order of the electroweak scale [38].

To derive the interactions among KK modes, one should start with the 5D Lagrangian, replace the 5D fields by their KK decomposition, integrate over x^4, and replace the 5D coupling parameters by their 4D counterparts which are identified by inspecting the interactions among 0-modes. The interactions
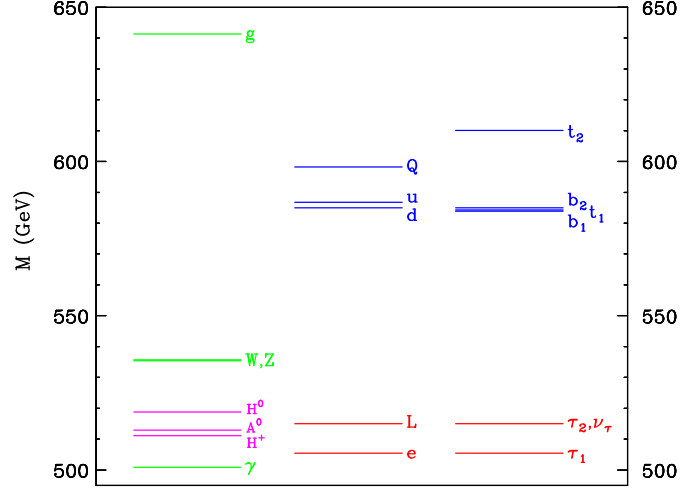
Fig. 10: Mass spectrum of level-1 KK modes for a compactification scale of $1/R = 500\,\text{GeV}$ and $\Lambda R = 20$, from Ref. [42].

Fig. 11: Feynman rules for vertices involving gluon KK modes ($G_\mu^{(j)a}$) and standard model particles. The couplings to quarks are chiral. G_μ^a is the massless gluon, $\mathcal{Q}^{(j)}$ are the $SU(2)_W$-doublet KK quarks, and $\mathcal{U}^{(j)}, \mathcal{D}^{(j)}$ are the $SU(2)_W$-singlet KK quarks.

of the KK modes with the massless gluon are fixed by the $SU(3)_c$ gauge invariance. For example, the coupling of a gluon to a pair of KK quarks is the same as for any quark pair, and the quartic coupling of two gluon 0-modes and two level-j KK gluons is the same as the quartic gluon coupling of usual QCD. However, the interactions of gluon KK modes with quarks are peculiar. As opposed to the 4D QCD interactions, which are vectorlike, the interactions of higher gluon modes distinguish between left- and right-handed quarks. This follows from the fact that the couplings of a spin-1 particle to fermions do not change the chirality, and thus a right-handed standard model quark couples only to the right-handed components of its KK modes (which are $SU(2)_W$ singlets, $\mathcal{U}^{(j)}$ or $\mathcal{D}^{(j)}$), while a left-handed standard
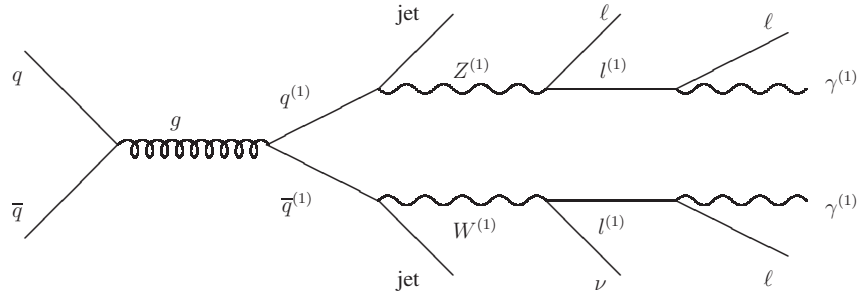
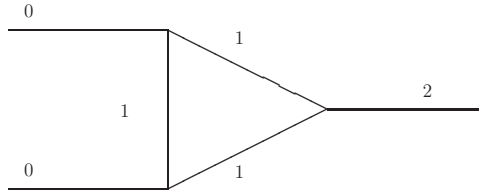**Fig. 12:** $3\ell + E\!\!\!/_T$ signal from UED. $\gamma^{(1)}$ is the dark matter candidate which escapes the detector.

Fig. 13: One-loop induced coupling of a 2-mode to two zero-modes.

model quark couples only to the left-handed components of its KK modes ($\mathcal{Q}^{(j)}$). These interactions are contained in the following terms of the 4D Lagrangian:

$$g_s G_\mu^{(j)a} \left(\overline{\mathcal{Q}}_L^{(j)} \gamma^\mu T^a (u_L, d_L)^\top + \overline{\mathcal{U}}_R^{(j)} \gamma^\mu T^a u_R + \overline{\mathcal{D}}_R^{(j)} \gamma^\mu T^a d_R + \text{H.c.} \right) , \qquad (44)$$

where g_s is the QCD gauge coupling. The Feynman rules describing the interactions of gluon KK modes to 0-modes are shown in Fig. 11.

The 1-modes may be produced in pairs at colliders. At the Tevatron and the LHC, pair production of the colored KK modes has large cross sections [43, 44] as long as $1/R$ is not too large. The colored KK modes suffer cascade decays [45] like the ones shown in Fig. 12. Note that at each vertex the KK-number is conserved, and the $\gamma^{(1)}$ escapes the detector. The signal is $\ell^+ \ell^- \ell^\pm + 2j + E\!\!\!/_T$. However, the approximate degeneracy of the KK modes implies that the jets may be relatively soft, and it could be challenging to distinguish them from the background. The leptons are also soft (with energies of a few percent of the compactification scale, as can be seen from the mass differences displayed in Fig. 10), but usually pass some reasonably chosen cuts.

A search for KK modes in events with two muons of same charge, performed by the D0 Collaboration [46], has set a limit of $1/R > 260$ GeV at the 95% CL in the minimal UED model. Using data collected by the CDF Collaboration during the Run I of the Tevatron, Ref. [47] has set a limit of $1/R > 280$ GeV based on the $3\ell + E\!\!\!/_T$ signature. Very recently, the ATLAS Collaboration [48] has searched in 1 fb^{-1} of data for the KK modes of UED using finals states with jets and $E\!\!\!/_T$, and has set a strong limit of $1/R > 600$ GeV. Larger LHC data sets can improve this limit, or alternatively, may lead to a discovery.

If a signal is seen at the LHC, then it is important to differentiate the UED models from alternative explanations, such as superpartner cascade decays [45]. Measuring the spins at the LHC would provide an important discriminant, but such measurements are challenging [44, 50]. A more promising way is to look for second level KK modes. These can be pair produced as the first level modes. However, unlike the first level modes, the second level modes may decay into standard model particles. Such decays occur at one loop, via diagrams having the structure shown in Fig. 13.
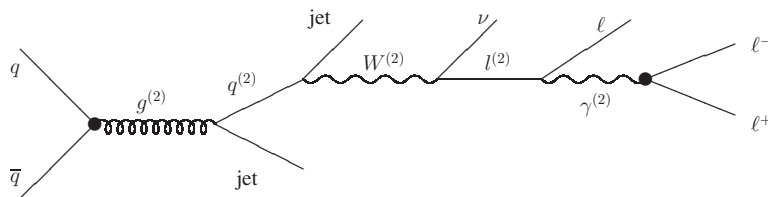
Fig. 14: s-channel production of the level-2 gluon followed by cascade decay, and $\gamma^{(2)}$ decays to e^+e^- and $\mu^+\mu^-$. The • represents an 1-loop effective vertex, as in Fig. 13.
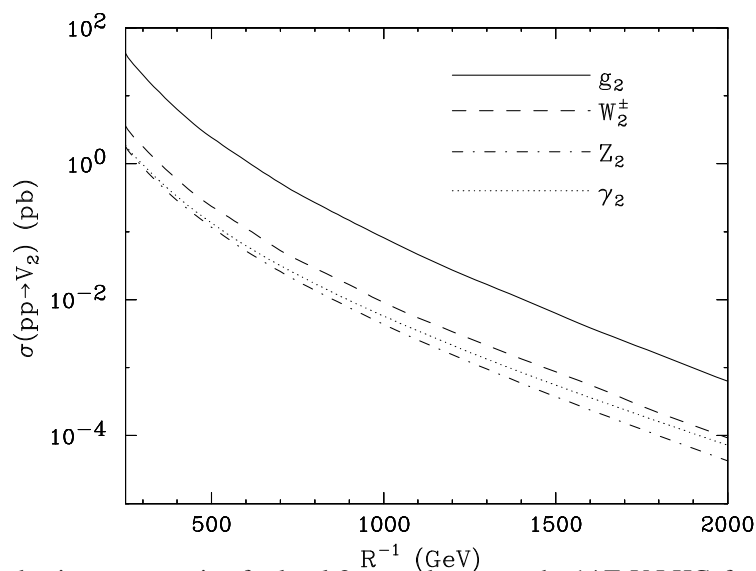

Fig. 15: Production cross section for level-2 gauge bosons at the 14 TeV LHC, from Ref. [50].

This diagram demonstrates that in the presence of loop corrections, the selection rule for KK numbers of the particles interacting at a vertex becomes

$$j_1 \pm \ldots \pm j_p = 0 \bmod 2 \ . \tag{45}$$

This implies the existence of an exact Z_2 symmetry: the KK parity $(-1)^j$ is conserved. Its geometrical interpretation is invariance under reflections with respect to the middle of the $[0, L]$ interval. Given that the lightest particle with j odd is stable, the $\gamma^{(1)}$ is a promising dark matter candidate. For $1/R$ in the 0.5 to 1.5 TeV range the $\gamma^{(1)}$ relic density fits nicely the dark matter density [49]. This whole range of compactification scales will be probed at the LHC [45].

Another consequence of the loop-induced coupling of a 2-mode to two zero-modes is that the 2-mode can be singly produced in the s-channel. The typical signal will be the cascade decay shown in Fig. 14, followed by $\gamma^{(2)}$ decay into hard leptons. The cross section in this channel at the LHC is substantial for compactification scales up to 2 TeV (see Fig. 15).

4.2.3 *More dimensions*

Gauge theories in more than four spacetime dimensions are nonrenormalizable. This is not a problem as long as there is a range of scales where the higher-dimensional field theory is valid. For gauge couplings of order unity, as in the Standard model, the range of scales is of the order of $(4\pi)^{2/n}$, so that only low values of n are interesting. Furthermore, the low energy observables get corrections from loops with KK modes. The leading corrections are finite in the $n = 1$ case and logarithmically divergent for $n = 2$, while for $n \geq 3$ they depend quadratically or stronger on the cut-off. Therefore, the effects of the unknown physics above the cut-off scale can be kept under control only for $n = 1$ and $n = 2$.

The case of two universal extra dimensions has been analyzed less extensively compared to $n = 1$ UED. The general features of the standard model in $n = 2$ UED are presented in Ref. [52]. The hadron collider phenomenology of $(1,0)$ modes, which are the lightest KK particles, has been explored in Ref. [53]. Cascade decays of spinless adjoints proceed through tree-level 3-body decays involving leptons as well as one-loop 2-body decays involving photons. As a result, spectacular events with as many as six charged leptons, or one photon plus four charged leptons are expected to be observed at the LHC. Unusual events with relatively large branching fractions include three leptons of same charge plus one lepton of opposite charge, or one photon plus two leptons of same charge.

The cascade decays of the $(1,1)$ modes [52], which are heavier than the $(1,0)$ modes by a factor of $\sqrt{2}$, generate a series of closely-spaced narrow resonances in the $t\bar{t}$ invariant mass distribution.

5 New particles, one or two at a time

There may exist various types of new particles, including vectorlike quarks or leptons, new gauge bosons (G', Z', W', γ'), new scalars (neutral or charged 'Higgs' bosons; diquarks, leptoquarks; color-octets transforming under $SU(2)_W$ as singlets, doublets, or triplets), or even more exotic ones. For illustration, in Section 5.1 we discuss spin-1 electrically-neutral bosons, usually known as Z' bosons, in a rather model-independent way. A similar approach may be applied to any other new particle, assuming that it can be produced and studied somewhat in isolation of other new particles (unlike the superpartners of KK modes of UED which are produced in cascade decays with many other new particles).

5.1 Z' boson

The couplings of a Z' boson to the first-generation fermions are given by

$$Z'_\mu \left(g_u^L \, \overline{u}_L \gamma^\mu u_L + g_d^L \, \overline{d}_L \gamma^\mu d_L + g_u^R \, \overline{u}_R \gamma^\mu u_R + g_d^R \, \overline{d}_R \gamma^\mu d_R + \ g_\nu^L \, \overline{\nu}_L \gamma^\mu \nu_L + g_e^L \, \overline{e}_L \gamma^\mu e_L + g_e^R \, \overline{e}_R \gamma^\mu e_R \right) ,$$

(1)

where u, d, ν and e are the quark and lepton fields in the mass eigenstate basis, and the coefficients g_u^L, g_d^L, g_u^R, g_d^R, g_ν^L, g_e^L, g_e^R are real dimensionless parameters. If the Z' couplings to quarks and leptons are generation-independent, then these seven parameters describe the couplings of the Z' boson to all Standard Model fermions. More generally, however, the Z' couplings to fermions are generation-dependent, in which case Eq. (1) may be written with generation indices $i, j = 1, 2, 3$ labeling the quark and lepton fields, and with the seven coefficients promoted to 3×3 Hermitian matrices (e.g., $g_{eij}^L \, \overline{e}_L^i \gamma^\mu e_L^j$, where e_L^2 is the left-handed muon, etc.).

These parameters describing the Z' boson interactions with quarks and leptons are subject to some theoretical constraints. Quantum field theories that include a heavy spin-1 particle are well behaved at high energies only if that particle is a gauge boson associated with a spontaneously broken gauge symmetry. Quantum effects preserve the gauge symmetry only if the couplings of the gauge boson to fermions satisfy anomaly cancellation conditions. Furthermore, the fermion charges under the new gauge symmetry are constrained by the requirement that the quarks and leptons get masses from gauge-invariant interactions with Higgs doublets or whatever else breaks the electroweak symmetry.

The relation between the couplings displayed in Eq. (1) and the gauge charges z_{fi}^L and z_{fi}^R of the fermions $f = u, d, \nu, e$ involves the unitary 3×3 matrices V_f^L and V_f^R that transform the gauge eigenstate fermions f_L^i and f_R^i, respectively, into the mass eigenstates. In addition, the Z' couplings are modified if the new gauge boson in the gauge eigenstate basis ($\tilde{Z}'_\mu$) has a kinetic mixing $(-\chi/2)B^{\mu\nu}\tilde{Z}'_{\mu\nu}$ with the hypercharge gauge boson B^μ (due to a dimension-4 or 6 operator, depending on whether the new gauge symmetry is Abelian or not), or a mass mixing $\delta M^2 \tilde{Z}^\mu \tilde{Z}'_\mu$ with the linear combination ($\tilde{Z}_\mu$) of neutral bosons which has same couplings as the Standard Model Z^0. Both the kinetic and mass mixings shift the

mass and couplings of the Z boson, such that the electroweak measurements impose upper limits on χ and $\delta M^2/(M_{Z'}^2 - M_Z^2)$ of the order of 10^{-3} [2]. Keeping only linear terms in these two small quantities, the couplings of the mass-eigenstate Z' boson are given by

$$g^L_{f\,ij} = g_z V^L_{fii'} z^L_{fi'} (V^L_f)^\dagger_{i'j} + \frac{e}{c_W} \left(\frac{s_W \chi M_{Z'}^2 + \delta M^2}{2s_W \left(M_{Z'}^2 - M_Z^2 \right)} \sigma^3_f - \epsilon Q_f \right) , \qquad (2)$$

$$g^R_{f\,ij} = g_z V^R_{fii'} z^R_{fi'} (V^R_f)^\dagger_{i'j} - \frac{e}{c_W} \epsilon Q_f , \qquad (3)$$

where g_z is the new gauge coupling, Q_f is the electric charge of f, e is the electromagnetic gauge coupling, s_W and c_W are the sine and cosine of the weak mixing angle, $\sigma^3_f = +1$ for $f = u, \nu$ and $\sigma^3_f = -1$ for $f = d, e$, and

$$\epsilon = \frac{\chi \left(M_{Z'}^2 - c_W^2 M_Z^2 \right) + s_W \delta M^2}{M_{Z'}^2 - M_Z^2} . \qquad (4)$$

A simple origin of a Z' boson is a new $U(1)'$ gauge symmetry. In that case, the matricial equalities $z^L_u = z^L_d$ and $z^L_\nu = z^L_e$ are required by the $SU(2)_W$ gauge symmetry. Given that the $U(1)'$ interaction is not asymptotically free, the theory may be well-behaved at high energies (for example, by embedding $U(1)'$ in a non-Abelian gauge group) only if the Z' couplings are commensurate numbers, *i.e.* any ratio of couplings is a rational number. Satisfying the anomaly cancellation conditions (which include an equation cubic in charges) with rational numbers is highly nontrivial, and in general new fermions charged under $U(1)'$ are necessary.

Consider first the case where the couplings are generation-independent (the V_f matrices then disappear from Eq. (3), so that there are five commensurate couplings: g^L_q, g^R_u, g^R_d, g^L_l, g^R_e. Four sets of charges are displayed in Table Charge, each of them spanned by one free parameter, x [54]. The first set, labelled $B - xL$, has charges proportional to the baryon number minus x times the lepton number. These charges allow all Standard Model Yukawa couplings to a Higgs doublet which is neutral under $U(1)_{B-xL}$, so that there is no tree-level $\tilde{Z} - \tilde{Z}'$ mixing. For $x = 1$ one recovers the $U(1)_{B-L}$ group, which is non-anomalous in the presence of one "right-handed neutrino" (a chiral fermion that is a singlet under the Standard Model gauge group) per generation. For $x \neq 1$, it is necessary to include some fermions that are vector-like (*i.e.* their mass terms are gauge invariant) with respect to the electroweak gauge group and chiral with respect to $U(1)_{B-xL}$. In the particular cases $x = 0$ or $x \gg 1$ the Z' is leptophobic or quark-phobic, respectively.

The second set, $U(1)_{10+x\bar{5}}$, has charges that commute with the representations of the $SU(5)$ grand unified group. Here x is related to the mixing angle between the two $U(1)$ bosons encountered in the $E_6 \rightarrow SU(5) \times U(1) \times U(1)$ symmetry breaking patterns of grand unified theories. This set leads to $\tilde{Z} - \tilde{Z}'$ mass mixing at tree level, such that for a Z' mass close to the electroweak scale, the measurements at the Z-pole require some fine tuning between the charges and VEVs of the two Higgs doublets. Vector-like fermions charged under the electroweak gauge group and also carrying color are required (except for $x = -3$) to make this set anomaly free. The particular cases $x = -3, 1, -1/2$ are usually labelled $U(1)_\chi$, $U(1)_\psi$, and $U(1)_\eta$, respectively. Under the third set, $U(1)_{d-xu}$, the weak-doublet quarks are neutral, and the ratio of u_R and d_R charges is $-x$. For $x = 1$ this is the "right-handed" group $U(1)_R$. For $x = 0$, the charges are those of the E_6-inspired $U(1)_I$ group, which requires new quarks and leptons. Other generation-independent sets of $U(1)'$ charges are given in Ref. [55].

In the absence of new fermions charged under the SM gauge group, the most general generation-independent charge assignment is $U(1)_{q+xu}$, which is a linear combination of hypercharge and $B - L$. Many other anomaly-free solutions exist if generation-dependent charges are allowed. Table ChargeGen shows such solutions that depend on two free parameters, x and y, with generation dependence only in the lepton sector, which includes one right-handed neutrino per generation. The charged-lepton masses may be generated by Yukawa couplings to a single Higgs doublet. These are forced to be flavor diagonal

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by the generation-dependent $U(1)'$ charges, so that there are no tree-level flavor-changing neutral current (FCNC) processes involving electrically-charged leptons. For the "leptocratic" set, neutrino masses are induced by operators of high dimensionality that may explain their smallness [6].

If the $SU(2)_W$-doublet quarks have generation-dependent $U(1)'$ charges, then the mass eigenstate quarks have flavor off-diagonal couplings to the Z' boson (see Eq. (1), and note that $V_u^L \left(V_d^L\right)^\dagger$ is the CKM matrix). These are severely constrained by measurements of FCNC processes, which in this case are mediated at tree-level by Z' boson exchange [57]. The constraints are relaxed if the first and second generation charges are the same, although they are increasingly tightened by the measurements of B meson properties. If only the $SU(2)_W$-singlet quarks have generation-dependent $U(1)'$ charges, there is more freedom in adjusting the flavor off-diagonal couplings because the $V_{u,d}^R$ matrices are not observable in the Standard Model.

The anomaly cancellation conditions for $U(1)'$ could be relaxed only if at scales above $4\pi M_{Z'}/g_z$ there is an axion which has certain dimension-5 couplings to the gauge bosons. However, such a scenario violates unitarity unless the quantum field theory description breaks down at a scale near $M_{Z'}$ [58].

Z' bosons may also arise from larger gauge groups. These may be orthogonal to the electroweak group, as in $SU(2)_W \times U(1)_Y \times SU(2)'$, or may embed the electroweak group, as in $SU(3)_W \times U(1)$ [59]. If the larger group is spontaneously broken down to $SU(2)_W \times U(1)_Y \times U(1)'$ at a scale $v_\star \gg M_{Z'}/g_z$, then the above discussion applies up to corrections of order $M_{Z'}^2/(g_z v_\star)^2$. For $v_\star \sim M_{Z'}/g_z$, additional gauge bosons have masses comparable to $M_{Z'}$, including at least a W' boson [59]. If the larger gauge group breaks together with the electroweak symmetry directly to the electromagnetic $U(1)_{\rm em}$, then the left-handed fermion charges are no longer correlated ($z_u^L \neq z_d^L$, $z_\nu^L \neq z_e^L$) and a $Z'W^+W^-$ coupling is induced.

The couplings shown in Eq. (1) lead to Z' production in the s-channel at colliders, and to Z' decays to fermion pairs. The decay into e^+e^- has a width

$$\Gamma\left(Z' \to e^+ e^-\right) \simeq \left[\left(g_e^L\right)^2 + \left(g_e^R\right)^2\right] \frac{M_{Z'}}{24\pi} . \tag{5}$$

The decay width into $q\bar{q}$ is similar, except for an additional color factor of 3, QCD radiative corrections. Fermion mass corrections are important for the $t\bar{t}$ final state. Thus, one may compute the Z' branching fractions in terms of the couplings of Eq. (1), but other decay channels, such as WW or a pair of new particles, could also have large widths that should be added to the total decay width.

Z' bosons with couplings to quarks (see Eq. (1) may be produced at hadron colliders in the s channel, and would show up as resonances in the invariant mass distribution of the decay products. The cross section for producing a Z' boson at the LHC which then decays to some $f\bar{f}$ final state takes the form

$$\sigma\left(pp \to Z'X \to f\bar{f}X\right) \simeq \frac{\pi}{48\,s} \sum_q c_q^f\, w_q\!\left(s, M_{Z'}^2\right) \tag{6}$$

for flavor-diagonal couplings to quarks. Here we have neglected the interference with the Standard Model contribution to $f\bar{f}$ production, which is a good approximation for a narrow Z' resonance. The coefficients

$$c_q^f = \left[\left(g_q^L\right)^2 + \left(g_q^R\right)^2\right] B(Z' \to f\bar{f}) \tag{7}$$

contain all the dependence on the Z' couplings, while the functions w_q include all the information about parton distributions and QCD corrections [54,55]. This factorization holds exactly to NLO, and the deviations from it induced at NNLO are very small. Note that the w_u and w_d functions are substantially larger than the w_q functions for the other quarks. Eq. (6) also applies to the Tevatron, except for changing the pp initial state to $p\bar{p}$, which implies that the $w_q(s, M_{Z'}^2)$ functions are replaced by some other functions $\bar{w}_q((1.96\,{\rm TeV})^2, M_{Z'}^2)$.

It is common to present results of Z' searches as limits on the cross section versus $M_{Z'}$. An alternative is to plot exclusion curves for fixed $M_{Z'}$ values in the $c_u^f - c_d^f$ planes, allowing a simple derivation of the mass limit within any Z' model (see *e.g.* Ref. [62]).

The observation of a dilepton resonance at the LHC would determine the Z' mass and width. A measurement of the total cross section would define a band in the $c_u^\ell - c_d^\ell$ plane. Angular distributions can be used to measure several combinations of Z' parameters (an example of how angular distributions improve the Tevatron sensitivity is given in Ref. [63]). Even though the original quark direction in a pp collider is unknown, the leptonic forward-backward asymmetry A_{FB}^ℓ can be extracted from the kinematics of the dilepton system, and is sensitive to parity-violating couplings. A fit to the Z' rapidity distribution can distinguish between the couplings to up and down quarks. These measurements, combined with off-peak observables, have the potential to differentiate among various Z' models [64]. For example, the couplings of a Z' boson with mass below $1.5\,\mathrm{TeV}$ can be well determined with $100\,\mathrm{fb}^{-1}$ of data at $\sqrt{s} = 14\,\mathrm{TeV}$. With this amount of data, the spin of the Z' boson may be determined for $M_{Z'} \leq 3\,\mathrm{TeV}$ [65], and the expected sensitivity extends to $M_{Z'} \sim 5 - 6\,\mathrm{TeV}$ for many models [66].

The Z' decays into e^+e^- and $\mu^+\mu^-$ are useful due to relatively good mass resolution and large acceptance. The Z' decays into $e\mu$ and $\tau^+\tau^-$, along with $t\bar{t}$, $b\bar{b}$ and jj which suffer from larger backgrounds, are also important as they probe various combinations of Z' couplings to fermions. The $pp \to Z'X \to W^+W^-X$ process may also be explored at the LHC, and is important for disentangling the origin of electroweak symmetry breaking. The Z' boson may be produced in this process through its couplings to either quarks [67] or W bosons [68].

6 Conclusions

So far the only robust evidence for physics beyond the standard model (defined as in Section 1) is that various astronomical and cosmological measurements require dark matter. This implies the existence of at least one new particle and some new symmetry that keeps it stable. It remains to be seen whether that particle has sufficiently strong interactions with the SM particles to be produced at the LHC or to be observed in direct detection experiments.

The Higgs sector is the least explored part of the standard model. There are hints of a Higgs boson at a mass of about $125\,\mathrm{GeV}$ from the ATLAS and CMS experiments, as well as from the Tevatron. Many precise measurements are required before assessing whether that is indeed the SM Higgs boson. If it will turn out to be a Higgs boson with non-standard interactions, then an important window to new physics will be opened.

Besides theories that address the hierarchy problem and include a dark matter candidate (such as the MSSM), there are many possibilities for new particles. Among those there are spin-0 bosons not necessarily associated with extended Higgs sectors (such as color-octets and lepto-quarks), spin-1/2 fermions (such as vectorlike quarks or leptons), spin-1 bosons (*e.g.*, Z' and W' bosons arising from extensions of the electroweak symmetry, or colorons arising from extensions of QCD), and higher spin particles. Each of these particles would lead to a variety of signals at colliders, so that the LHC experiments should search for these in as many final states as possible.

It should also be emphasized that physics at the TeV scale could involve new strong interactions, and that some if not all SM particles may be composite. Given the current poor understanding of strongly coupled field theory, the measurement of various differential cross sections at the LHC could uncover truly unexpected phenomena.

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